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THE MONIST

THE PRODUCTION OF PSYCHIC STATES.

WHEN one takes a comprehensive glance over the field of the philosophy of mind, he cannot but be amazed at the number of gaps that need to be filled and the number of fundamental problems that psychologists have not yet attacked. Among these, the problem of how consciousness originated has only been touched upon; the general function of conscious activity as a biological adaptation demands much clarification. Psychologists have delayed the study of such problems when they have worked exclusively from the presuppositions of behaviorism or psycho-physical parallelism. The behaviorists rightly have put man's actions into the order of nature and denied that a supernatural entity, consciousness, imposes its will on the direction of natural forces. And yet they have missed the full profit of their criticism of the older psychology by failing to see that mental states may have a place in the natural order itself. Parallelism is accepted by many psychologists as a working hypothesis, and as such it has done good service. And yet parallelism, erected into a metaphysical theory, by its very inconceiveability is likely to degenerate into epiphenomenalism, and epiphenomenalism to be replaced by radical behaviorism.

Philosophers also have contributed toward delaying the solution of the psycho-physical problem. The conflict between monists and dualists has deferred consideration of many important aspects of that problem. The strength of metaphysical dualism (animism) is chiefly negative, and the natural desire of man to believe that he has a substantial soul is balanced by a desire to conceive the universe as one.

if possible. We may grant that the issue between monism and dualism is the most important one in the field of cosmology, but in attempting to contribute toward a solution of the problem, it would seem that indirect procedure is more hopeful than direct attack. Now dualism in its present stage of progress is most useful in its *criticism* of monistic theories. At the time *Body and Mind*[1] was written, Professor McDougall made a telling record of these criticisms. Some of them have been answered. In answering them, however, the method employed has been not so much to reconsider the fundamental dispute between monism and dualism, as to seek a consistent monistic position. May we not say, therefore, that it is of the greatest importance now to work the monistic hypothesis for all that it is worth in an endeavor to see whether it will meet all of the difficulties advanced by the dualists?

One step taken in this direction is highly significant, for it nullifies one of dualism's most biting criticisms. Dualists have stressed the difficulties involved in the idealistic position that matter is a form of consciousness. Dr. Strong's searching analysis in his recent book, *The Origin of Consciousness*,[2] has shown that consciousness is *adventitious* to psychic states. Thus, if psycho-physical monism be taken as the best of the monistic theories, the demand no longer exists to explain how matter is a form of consciousness; the problem is shifted to the question: how are we to conceive as one substance matter and the psychic states to which consciousness may, under suitable circumstances, be adventitious?

Another problem that has beset monism in the past is the difficulty of explaining the unity of consciousness. To this question Dr. Strong has addressed himself and made an admirable contribution. In a recent paper[3] I have

[1] London, 1911.
[2] London, 1918.
[3] "The Unity of Consciousness," *Jour. of Phil.*, Vol. 18, pp. 347-357.

endeavored to show that the unifying principle need not, as hitherto supposed, be sought either among psychic states or among sensory neural processes, but may be found in the motor neural processes which govern attention.

Naturally the epistemological problem has been much discussed in this connection. And here the solution most congenial to psycho-physical monism is that of epistemological dualism. The plea of epistemological dualism against objectivism runs: Interpret man's behavior, but do not despise conscious activity. Realize that man will not endure the repudiation of his thoughts and emotions and desires. Find a more complete way of describing his behavior, a way that will include his conscious states.

With the adoption of such a standpoint, the problem of how consciousness originated is recognized, taking the place of a fictitious problem of how a *myth* of conscious activity arose or of a dogmatic assumption of epiphenomenal states with a multitude of unaccounted for qualitative differences. Dr. Strong has made a beginning in his attempt to solve certain difficulties in the way of conceiving mind as a product of evolution. Even if Dr. Strong's conclusions should be accepted in their entirety, however, it is plain that the study of the genesis and development of consciousness are just begun. The present paper is an attempt to contribute one step toward the understanding of how consciousness and psychic states function in human behavior. The particular field of investigation chosen is that of the relation of sensation, feeling, and emotion to their neural bases.

I. A FUNCTIONAL VIEW OF NEURAL ACTIVITY.

§1. In a recent paper,[4] I have maintained that cognition and feeling are independent aspects of conscious activity.

[4] "The Coördinate Character of Feeling and Cognition," *Jour. of Phil.*, Vol. 18, pp. 288-295.

The first assumption of the present paper is that the same independence does not extend to the neural bases of cognition and feeling. I call this an assumption, for it is *conceivable* that a *differentia* of neural substance concomitant with a qualitative conscious difference might be discovered for each of the multitude of conscious qualities. Now science conceives the physical world in terms of a dynamic, uniform substance, organized in a great variety of ways. The neural concomitants of conscious states, therefore, are to be sought in characteristic types of order. Here a choice presents itself: is the order static or dynamic or both? The assumption that I have made asserts that we are not to seek for coördinate static organizations of neural substance concomitant with feeling and cognition respectively. On the cognitive side we do indeed find a limited static differentiation in the case of the sense end-organs and cortical areas. This, however, is not sufficient in itself to account for the diverse cognitive qualities. Some dynamic, or, to use a better term here, *functional* differentiation must also be assumed. But when we come to consider the neural basis of feeling, we do not find specific sense end-organs. It is natural, therefore, to conclude that the neural basis is functional, rather than structural. So far as I know, all the current theories of feeling are in accord with this hypothesis with the exception of Professor Titchener's theory that the free, afferent nerve-endings are organs of feeling—a theory which is not widely accepted. The second assumption that I make is that no sense end-organs of feeling will be discovered.

The two assumptions that I have made are a result of the survey of progress attained in the study of feeling and emotion. The diffuse character of their physiological bases is everywhere asserted. James' theory would make certain sensations the bodily causes of emotions, sensations which have no special emotional character. Dewey's theory

emphasizes the conative aspect of emotions, but here again the emotional character is not specifically localized. Marshall, in his nutritive theory, finds no definite feeling end-organs. Other theories might be cited to support our assumptions. It is not assumed by these theories, however, that feeling and cognition are not coördinate so far as consciousness is concerned. One may give full credence to Titchener's view of the latter question and to the recent results of Wohlgemuth, without claiming in addition that the neural bases of feeling and emotion are more than functional.

§2. A functional view of neural, as related to conscious activity, points the way to a new method of approach to the psycho-physical problem. There has been a general movement away from 'faculty psychology.' It is now generally recognized that consciousness is one in its functioning, and that the different processes into which a conscious moment may be resolved are but distinct *aspects* of a unified process. The spirit of faculty psychology, however, still haunts the systematic psychologists, not in their consideration of conscious activity, but in their consideration of neural processes. Rather than to try to distinguish neural bases of feeling and cognition, will it not be more profitable to consider the conditions of the production of these aspects of conscious activity? Especially significant will be a consideration of their production in reference to attention and to marginal elements of consciousness. I purpose, therefore, to discuss the conditions under which neural processes function now as cognitive, now as feeling-concomitants.

II. THE PRODUCTION OF PSYCHIC STATES IN POINT OF ORIGIN.

§4. The first subject of inquiry relates to the function of psychic states as respects their origin. I may begin by noting a difference in the evolutionary theories of cognition

and feeling. Cognition is explained as a useful means of integrating the sensory and motor halves of the reflex arc when the stimulations are various and conflicting and tend in incompatible directions of motor expression. Thus cognition is conceived to be a useful adaptation at the very moment of origin. With feeling the case is entirely different. The biological function of feeling is conceived to be that of encouraging the individual to persist in beneficial activities and to desist from harmful ones. Here the adaptation implies past experience; feeling is useful in planning action based on the past. It is thus conceived not as useful at the moment of origin, but in the course of experience. As to origin, it is regarded as a chance variation which persisted in the first instance because it was not fatal to the existence of the organism. Needless to say, the furtherance-hindrance theory does not ascribe a survival-value to feeling *at the moment of origin.*

The question, therefore, is suggested: shall we seek further for a survival-value in point of origin? (I present this question because I have in view a description of the functions of cognition and feeling of a mature individual with respect to their biological value. The instrumentalists have laid stress on the instrumental character of the reflective judgment, but Professor Moore[5] refuses to commit them to a more general belief in the instrumental character of all the aspects of consciousness. Now the discussion of mature feeling and cognition raises the question of the place of immediate values in an evolutionary theory.) As to whether feeling is useful in point of origin. I can only say that when we regard its conscious character, feeling at the moment of origin seems to be of contributory value only in increasing the range of contact of the organism with its environment, but I hope to show that when we regard the production of feeling, we find that feeling and cognition alike are comprehended in a unified neural function whose highest point is conscious attention.

[5] *Jour. of Phil.*, Vol. 17, pp. 514-519.

§4. At this point, however, I would dispel a possible misconception arising from the treatment of conscious functions as adaptive. The disputes that linger over this field would vanish if a certain distinction were more consistently made. The whole notion of adaptation admits of only one immediate value. This value is the biological end of evolution, survival-value. All structures and functions that persist without being explained in terms of survival-value, although without survival-value, are not harmful to the organism. The biological treatment is from the standpoint of an observer of the evolutionary process. Now it would be utterly false to imagine that this standpoint does away with intrinsic values. The latter, as well as some instrumental values, subsist from the standpoint of the individual organism. This second standpoint arises in connection with an individual consciousness, and should not be confused with that of an observer of the evolutionary process. Elsewhere I have made this distinction in more detail.[6]

We have, therefore, two centers from which valuations may be made, the consciousness of an observer and the consciousness of an individual. From the individual's point of view, the biological end of value is often swept aside for the sake of values that have arisen, presumably, as accidental by-products of the evolution-process. Thus a martyr, intent on the propagation and defence of ideal, intrinsic values will utterly disregard the "lower," naturalistic end of life with its fundamental instinct of self-preservation. He has the right to argue that the ultimate values of the individual are something quite apart from the original biological function which the structures upon which they depend subserve. He is quite within his right in refusing to accept mediaeval acquiescence in the doctrine that the life-preserving instincts should be exercised only in view of

[6] *Cf. Values, Immediate and Contributory*, New York, 1920, pp. 31-36, 104-112.

their biological end. Mature conscious activity, from the individual's point of view, may be expected to exhibit many instrumental features, especially in the *acts* of consciousness, but in respect to *content* we may expect to find much that transcends the instrumental.

When, however, we consider the origin and function of consciousness from the biological standpoint, we shall seek, from the standpoint of naturalism, instrumental values wherever they may be found. It is my purpose to show how feeling and emotion may be conceived to have usefulness in helping to adjust the individual to his environment.

§5. If, as Dr. Strong argues,[7] consciousness is adventitious to the psychic state, a prior question to that of the origin of consciousness will be: what is the origin of psychic states? We are helped here by the fact of experience that many of the steps of reasoning are conducted on a subconscious or unconscious level, so that it is reasonable to suppose that the gap between a conscious memory and the condition of the brain that subserves the memory is not so wide as has been thought. Since many stimulations are received and transmitted to the brain, but are cognized only after the stimuli have been withdrawn, we must believe that the brain is often near a psychic condition even though the stimulations do not at the time issue in *givenness*. On the border-line between psychic states that are given and psychic states that are not given are objects in the fringe or margin of consciousness. I have argued elsewhere[8] that their behavior in respect to attention indicates that pleasure and displeasure are marginal in nature. I would state further that I believe that welcomeness or unwelcomeness are the conscious indications of incompletely developed psychic states that are positvely or negatively instrumental to the welfare of the organism.

[7] *Op. cit.*, pp. 194-209.
[8] *Jour. of Phil.*, Vol. 18, pp. 288-295.

The theory of psycho-physical monism seeks to close the gap between conscious states and psychic states, on the one hand, and between psychic states and their neural concomitants, on the other. It proclaims the continuity of substance throughout the development of the organism. If consciousness is adventitious to psychic states, we are delivered from the extravagance of the theory that all nature is conscious. If the aspects of psychical activity are functions of neural states, brain-processes in respective functional activities *are* sensations and emotions and feelings. The advantages of psycho-physical monism in a description of the interrelation of the various psychic functions are obvious and need not be refined upon here.

III. THE PRODUCTION OF PSYCHIC STATES IN MATURE CONSCIOUS ACTIVITY.

§6. Our problem is that of investigating the production of feelings and cognitions with special reference to their usefulness to the organism. We shall ask: What is the biological significance of the ideational and emotional elements that go to make up a conscious moment? The answer to this question, so far as the ideational element at the focal point of attention is concerned, is clear. The motor factors of attention are the means of integrating diverse stimulations, and the idea (identical under our theory with its neural basis) is thus biologically useful in giving directions to eventual action. As Bergson would say, our perceptions mark out lines of possible activity of the organism in respect to its environment.

The conditions under which the marginal elements of consciousness are produced are understood on the cognitive side. Whether these elements be the result of present stimulations or the recurrence of images in memory, their presence is conditional upon their belonging to simulta-

neous movement systems.[9] Their biological value to the organism is apparent, increasing, as they do, the probability of useful reaction to present stimuli.

The conditions under which feelings and emotions are produced and their relation to the ideational elements of consciousness, however, have not received the attention they deserve. Here the same two possibilities present themselves as occur in the case of the ideational elements; they may be the outcome of present stimulation, or they may, in some way, be traceable to past experience.

§7. It is necessary at this point to speak of the one theory that would connect certain present feeling-states with past feeling-states, the theory, namely, of "affective memory." A careful survey of the literature bearing on the subject convinces me that the existence or non-existence of such a process cannot yet be demonstrated with certainty. I much suspect, however, that "affective memory" rests on false assumptions. The issue may be stated in the form of three questions: Is there in feeling a process corresponding to memory in the field of cognition? Can memory-images of pleasure and displeasure be recalled independently of the ideas to which they were previously attached? Is the feeling-tone of a remembered idea the one attached to it in the original experience, or is it a new one?

The first of these questions rests on the false assumption that cognition and feeling have coördinate and non-identical types of neural bases. We have made the point that although cognition and feeling are coördinate to introspection, this does not demand that they have different types of neural bases. It is not surprising that feeling is coördinate with cognition in consciousness in view of the fact that qualitatitve differentiation is the predominant characteristic of psychic states. But why should we

[9] *Cf. Movement and Mental Imagery*, M. F. Washburn, Boston and New York, 1916, pp. 130-131.

assume that this coördination extends to their neural bases? Only on the analogy suggested by the existence of sense end-organs and specific cortical areas. The force of this analogy, however, is lost when we consider that neural differentiation is very meager when compared with the differentiation of cognitive qualities.

The second question would appear to be as fruitless as the first. Pleasure is pleasure; displeasure is displeasure. The pleasure attached to one experience is not different from the pleasure attached to another experience. We have no clear evidence that there is such an entity as a *specific* affective-image; feeling appears rather to be *generic* in character. As to the third question, it may be repeated that the assumption is made that there are specific differences of pleasure and displeasure.

Our own theory puts the question as to the existence of "affective memory" in a new light. Pleasure and displeasure are marginal in character; direct attention to them causes them only to vanish and to be replaced by cognitive elements. On the assumption that they have a diffuse neural basis, it is suggested that they may present to consciousness the welcomeness or unwelcomeness to the organism of neural states that have not yet reached a degree of activity where they function cognitively, but are associated with the neural states that function as marginal cognitive elements of consciousness. If this be true, neural activity which functions psychically, will include besides the cognitional determinants certain neural states not sufficiently active to produce cognition, but sufficiently active to produce feeling. The feeling thus produced must be distinguished from the feeling-tone of a present sensation, whose neural basis is probably even more diffuse activity, due to "surplus excitation," the release of "stored energy," chemical changes in organic tissues, *etc.*—precisely which of these is not yet known. I will only suggest that this

theory would yield interesting results if applied to an analysis of the aesthetic experience.

§8. While it is maintained that the neural bases of cognition and feeling do not differ in character, in that a specific neural complex may function at one time as cognition, at another, as feeling, at any one conscious moment the neural bases of the cognitive and the feeling-elements will be different. If direct attention to feeling or emotion replaces it with cognitive elements which were not present when the emotion was felt, is it not reasonable to believe that the neural activity giving rise to the emotion is identical with that giving rise to the previous cognitive experience. This corollary of our theory applies to *any* surplus excitation.

§9. We are now in a position to discuss the significance of the *emotional* elements present in particular conscious moments. There seem to be three possibilities here. Professor Dewey emphasizes that emotion occurs as the result of a conflict of desires. Now it is impossible to attend to two impulses at once. Where attention fluctuates rapidly between two opposed tendencies to act, the emotion may well represent one of the ideas crowding in and endeavoring to replace the other in cognition. This may be called the first type of emotion.

Secondly, the affective aspect may be due to the partial arousal of neural processes that form the basis of past memories. To speak entirely inaccurately, yet suggestively, feelings and emotions here are on the way to becoming ideas. We may speak of this type as the substitution of emotions for ideas. Such occurs frequently when prejudices are aroused. It is a matter of common observation that ideas casually suggested will at times provoke violent emotional reactions. Persons who exhibit prejudice usually do so on scant rational grounds. Someone I know is prejudiced against the wearing of spats, but, when

pressed for a defence of his feeling, gave most inadequate reasons, saying that they do not sufficiently protect the feet, *etc.* I believe that the simplest explanation of prejudice is that old associations are partially aroused. Brought from the fringe of consciousness to full attention, they would, as memories, exhibit conflict between themselves and the idea suggested. Remaining as they do in the margin of consciousness, they serve only to tinge the present idea with a strong tone of displeasure or emotion.

§10. It would be absurd, however, to claim that emotions represent only memories of the past. Genetically, emotions antedate the particular ideas which explain their nature. The child cries before he attends to the object of his grief. The adolescent youth experiences sweet delights of whose biological cause he may be entirely ignorant. Some emotions have their meaning in the undercurrent of past experience, but others burst in upon us as a totally new type of experience. The latter emotions must be traced to inherited, rather than acquired, reactions. They are the conscious indications of the operation of instinctive processes. Their neural basis is inherited associations of cortical states. From the neural standpoint, however, this third type of emotional significance will not differ from the other two types.

IV. PRACTICAL CONSEQUENCES.

§11. In conclusion I would remark upon some of the practical consequences of the theory here outlined. I have forborne studiously to seek any compromise with or help from psycho-analytic teachings in the construction of my theory, preferring to base it wholly upon orthodox psychology. I do believe, however, that it affords opportunity for interpretation of some of the results of psycho-analysis. Here I wish to indicate what appear to be the consequences

of my position in respect to education, particularly in the spheres of ethics and religion.

Why is it that conviction is not obtained among average persons through the presentation of intellectual arguments so easily as through an appeal to the emotions? Why is the appeal to prejudice so potent to secure conviction? We must conclude that the appeal to emotion is wider than the appeal to specific reasons, for in emotion many associations are partially aroused which may join with the conscious idea in checking any attempt to thwart or contradict it. John Mill[10] recognized the importance of associations in explaining the action of conscience. May it not be true that where quick, decisive action is called for, especially from minds essentially unintellectual, the operation of conscience renders a service rather than creates a barrier to correct action?

Again, many philosophers and teachers of religion lay emphasis on the cultivation of feeling and emotion in connection with fullness of living and deepest religious experience. There is a strong predilection among the southern races to regard an emotional nature as more near to "reality" than cold, Anglo-Saxon intellection. The contemplation of an Absolute Idea or communion of man with God in the beatific vision is reserved for the perfection of human nature. Mortals of average intelligence build useful lives on the foundation of wholesome prejudices. In view of the limitations of attention, it would seem that the coöperation of emotion and feeling with ideation were devoutely to be desired rather than impatiently to be set aside. Furthermore, in the case of religious experience there may be experiences not possible to man through ideation alone, the elements of the experience being too many to come to attention at once and too rich to be comprehended in a general concept. Here emotion and feeling, with their

[10] *Utilitarianism*, Everyman ed., New York, 1910, p. 26.

broad range of reference, may serve a necessary and priceless purpose. Practically, then, we shall not reasonably seek to rid ourselves of all emotion, and by so doing lose in imagination and intuition, but we shall rather endeavor to separate the wheat from the chaff in emotions, and correct, rather than abandon prejudice.

The substance of this paper may be summarized as follows:

I. A FUNCTIONAL VIEW OF NEURAL ACTIVITY.

§1. Cognitive and feeling-states are coördinate. But assuming (*a*) that neural states are qualitatively much less differentiated than psychic states, and (*b*) that no specific sense end-organs of feeling will be discovered, we may not conclude that this coördination extends to their neural bases.

§2. If §1 contains a point well taken, it will be profitable to investigate the conditions of production of feeling and cognition, rather than to inquire closely into the structure of their neural bases. The neural difference may be functional, not structural.

II. THE PRODUCTION OF PSYCHIC STATES IN POINT OF ORIGIN.

§3. The origin of psychic states is a matter closely related to the biological (survival-) value of the different conscious processes.

§4. The instrumental value of the conscious processes considered from the standpoint of an observer must not be confused with valuation from the standpoint of the individual.

§5. The origin of psychic states is best explained by the theory of psycho-physical monism, according to which the gap between neural and psychic processes is closed.

III. THE PRODUCTION OF PSYCHIC STATES IN MATURE CONSCIOUS ACTIVITY.

§6. Although the biological function of the cognitive elements of consciousness is understood, that of the feeling —and emotional elements needs further investigation.

§7. "Affective memory," analyzed, proves to be based on the analogy denied in §1. The observations cited in its support find their explanation under the theory that the neural activities which function consciously are wider than those which produce cognitive elements. Feeling is produced when neural states of low intensity function consciously. This low degree of excitation may take place in areas previously aroused by stimulation, or it may be a surplus excitation of present sensations or the rudimentary form of organic sensations, *etc.*

§8. The same functional activity is never the basis of cognition and feeling at once.

§9. An emotion may represent the unattended to idea when two opposed ideas fluctuate in attention. *Or,* it may represent ideas associated with the idea in mind, which have not reached the ideational level of activity.

§10. *Or,* an emotion may be the conscious representative of an instinctive tendency felt for the first time.

IV. PRACTICAL CONSEQUENCES.

§11. Practically, the following consequences of the theory may be suggested:

(*a*) A way of reconciliation of the results of psychoanalysis with orthodox psychology is indicated.

(*b*) The utility of the education of conscience in the moral field is defended.

(*c*) Emotions may serve in religion to put the individual into contact with wider experiences than can be attained through intellection alone.

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THE ESSENCE OF RELIGION.[1]

ONE of the questions which cause both the deepest interest and the utmost disharmony amongst men at the present time, is undoubtedly that which deals with the status of religions and with their future. The most diverse and contradictory opinions are upheld with a degree of erudition and dialectical ability that seem to be the prerogative of no single school. And we are far indeed from being within measurable distance of a mutual understanding.

This extreme divergence in appreciation is due not only to the special nature of the subject, in judging which feeling claims to have a share: it is also due apparently, to the fact that, as a general rule, before discussing the permanence or the decline of religions, we fail to enquire of what it is that religion really consists, what is its essence. What is the meaning of the radical contradiction between the affirmations of the one side and the negations of the other, if neither is speaking of the same thing? When dealing with progress or with decline, is it not absolutely necessary, if we would refrain from judging by appearances, which are so frequently deceptive, that we seriously question ourselves as to the precise object whose degree of vitality we are endeavoring to discover?

Now, it must be recognized that the man who purposes to determine the essence of the religious phenomenon,

[1] [Authorized translation by Fred Rothwell.]

impartially and in a way that is valid to all, will find the problem a singularly difficult one.

The method which, at the outset, offers itself to the mind, consists in studying all existing religions and finding out what they have in common. The religions however that have been, and still are, practised by men, are innumerable; they present every imaginable and unimaginable variety. Many consist of scarcely anything but ceremonies; others almost taboo ceremonies. The latter are altogether *exterior,* the former altogether *interior.* Some recognize gods or a god; others are atheistic. In some, men have need of the gods; in others, it is the gods that have need of men. Some particular religion is mainly a code of morals; many religions are opposed to any idea of morals. Some gods are tolerant or accessible as regards other gods, some particular one, concentrating within himself all power and perfection, recognises no other god as his superior. There are religions that incite men to action, war and conquest; others recommend them to practise renunciation, abstention, and detachment from all things. Some promise immortality, others would have man kill out the desire to live and feel.

What have all these widely different forms in common, unless it be an almost empty concept, a word, a label? What interest is there in enquiring as to the destiny of a thing, the nature of which we cannot attempt to determine without the spirit of erudition at once intervening to prove that our field of observation, being too restricted, allows of no general conclusion being reached?

May we not, however, substitute for this purely logical and abstract method, the historical one, if we would be in a position to reach valid conclusions?

History sets up a fundamental difference between things: it distinguishes between those that are living and those that are dead. Democracy, science and art are living

things: human sacrifices and slavery, though they do exist here and there, belong to the past. History registers the changes that have taken place in the sum total of the conditions of human life. Institutions that are incompatible with the conditions of modern life, though they may seem to exist, are, in reality, but survivals of past forms of existence, and, as such, must disappear.

This is a very clear and plausible doctrine, the more so as it unites mankind with what is most familiar to us: the living being. Assuredly we have not yet discovered the way to bring the dead back to life. But the precise thing we have to consider is whether, in the spiritual domain, the words *dead* and *living* can be applied with the same certainty as in the domain of biology. I read in an article by William Knight, on the historical method in philosophy,[2] "Alike in philosophy and art, in social life, in politics and in religion, all the best things that are ever evolved are superannuated in time. They have to die and be reborn, in incessant palingenesis." Though it may not be possible to say that, in the spiritual order of things, birth necessarily calls for death, and death for rebirth, it is still true that progress frequently consists in recalling to existence, by adapting it to new ideas, some particular form of art or conception of life that seemed to have disappeared for ever. While the whole of the past is not destined to become present again, many shadowy forms, at all events, stretching their arms towards the shores of the future, have a chance to cross the stream, if men continue to be eager for novelty and change. The historian's assertion of the disappearance of a certain form of existence is not sufficient to prove its total and final extinction.

Another way of questioning history is to demand that it reveal to us the primary origins of institutions. Was not the oak previously formed in the acorn? Does not man

[2] *Hibbert Journal*, July, 1904, p. 756.

inherit from the child? The maxim that the water of a stream is purer at its source has certainly proved itself very productive of results. Still, though a singularly powerful principle in practice, is this maxim a safe guide in theory? As a matter of fact, the source eludes our search. The most remote prehistoric times show us works that are infinitely complex and derivative. So also the child, when beginning to speak, actually goes through processes which call into play all the principles of our logic. Again is it certain that human progress advances in a straight line and that the present proceeds from the past by mere development? The stone which the builders rejected, we read in the Gospel, has become the head of the corner. From the effort to meet the physical needs of man, science came into being, not by a process of development, but by accident: science is the disinterested investigation of truth. It is gratuitous to suppose that there is nothing more in present-day religion than in the taboo or the totem of primitive races. It may be that new elements have been added on to them by a process of epigenesis, and that in these elements is now to be found the center of religious activity. Protestantism was intended to be a reaction against change: it has become the very incarnation of change.

Finally, there is a third way of dealing with history, and that consists in determining the law by which the phenomenon under consideration evolves. By appropriate analyses and comparisons, we throw light upon the phases of this phenomenon, and the constant order according to which, here and there, they succeed one another. Henceforward, this very law constitutes the essence of the phenomenon; it permits to a certain extent, of our foreseeing the future in store for it: a definite and methodical march, like that which a man attributes to the course of his life, when he considers it from the point of arrival, a more or

less visionary march, all the same, lacking the sanction of confirmation, since the future is an unknown quantity. What striking contradictions the future, on becoming the present, inflicts on our most carefully established predictions! These mathematical curves in which we take delight are obtained by leaving out or by taking up divergent phenomena. Now, who can say that some one of these scattered forces, more or less restrained for a time, will not leap up victorious some day, like a spring that has been held down all to no purpose? The order in which moral phenomena succeed one another is not fortuitous, but it is contingent.

An ever increasing number of persons endeavor to add on to or substitute for the purely logical and the historic method, a strictly scientific one. The consequence is that we have two sciences: positive psychology and sociology, both of which endeavor to theorize about religious phenomena. In reality, the results at which these two sciences arrive are rather contradictory than complementary to each other; since, in the one instance, religion radiates from the individual consciousness towards social institutions, whereas in the other case, it has its origin in society and makes its way from without into the individual conciousness. These two methods of explanation, however, have this in common: they tend to break up and destroy that which they explain. Some explanations establish and justify: a master's authority is established by proving his capacity. There are, however, explanations that cause their object to vanish away: the explanation of the sense of initiative by suggestion reduces this sense to an illusion. Religion is the belief in a reality which transcends the laws and phenomena of our world. To prove that such a belief, exclusive of every object corresponding thereto, must of necessity result from the spontaneous play of the natural

psychic or social forces, is to destroy the legitimacy of this belief, and make it impossible in an enlightened mind.

If positive psychology and sociology are to keep their promise, then the question of the essence of religion is very easily solved: religion has no other essence than ignorance. And the real problem can only consist in endeavoring to discover those social or psychological conditions which induced man falsely to imagine that he had dealings with supernatural powers.

But it is advisable to recognise that science, here as elsewhere, is not something complete and existing in itself from all time, thus leaving man no other task than to contemplate it, once he has discovered it. For the last three centuries, there has been substituted for this ontological conception of science, the idea of a purely experimental science, which asks questions of nature though without prejudging the reply. Religious psychology and sociology demand of the human soul and society whether they contain within themselves the wherewithal to explain religious phenomena. Not *a priori* but by considering the results obtained by investigators can we know how far these sciences succeed in resolving religion into elements devoid of religious characteristics. Now, it can scarcely be disputed that hitherto only the externals, the ambiguous traits of religious phenomena have been, in a way, subjected to psychological or sociological explanation.

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We are compelled to face the problem of the essence of religion. To state this problem in its true terms, is to enquire whether it is possible, as regards religion, to transcend both fact and concept, and to attain to idea. It is not sufficient to know that religion has been this or that, in order to be in a position to conclude that it is to continue or to disappear. It may happen that it continues in a form

different from those it has hitherto presented. It may also be that it seems to continue, because certain indifferent forms of it remain, whereas in reality it is disappearing as regards all that more especially constituted its greatness.

In like fashion, we see that the knowledge of the simple concept of religion is but a trifling thing. What is called concept includes nothing else than the sum total of the necessary and adequate conditions of the existence of a thing. The concept of religion exhibits the characteristics found in every religion, present in the lowest and the highest alike. To content oneself with this concept in deciding whether religion subsists or is to subsist, is to regard existence, pure and simple, as adequate, without enquiring into its quality. What we should need to possess, in order to pronounce a judgment of any importance, is, strictly speaking, the idea of religion, the Platonic idea, as it were, the notion of what religion can and should be, if it is to exist in the truest and highest way possible.

It is of religion as thus understood, rather than of some one of its historical forms, that it is important for us to know if it exists at the present time, if it appears as though it must continue to exist and to act in the world. It is in this idea of religion that the essence we are seeking strictly consists.

Is it possible to determine such an idea? Are we not simply expressing opinions that are subjective, individual and worthless, when we venture to speak, not of what is, but of what may or ought to be? Is it possible to enter upon such a problem without leaving the plane of fact and reason, without plunging and losing oneself in the void, like the dove with which Kant amused himself? We must note that, both in everyday life, and in philosophical reflection, we have constantly to deal not with concept, but with idea. When we speak of the future of art and science, of democracy and socialism, we are not simply thinking of

them as actually given or presented. or as they would be defined in a logician's generalization: we assuredly have in mind the thought of what science or democracy can and ought to be, to attain to full realisation, *i.e.* not the concept but the idea of science or of democracy.

Is such an idea but a feeling or a desire, a fancy or an arbitrary invention? We must renounce proving its value by confronting it, purely and simply, with the facts of experience, as is done in testing scientific hypotheses: indeed, the idea in question, by definiton, transcends experience. The harmony, however, of our conceptions with the things of the outside world, is not the only means of control at our disposal. We may also consider the harmony of minds with one another. And, looking at the matter closely, the external criterion, afforded by the sensible reality, exists only as the application of the internal criterion, deduced from the common adhesion of minds. For how do we know that a thing may be regarded as existing apart from intellects, unless it is because the latter are agreed as to their way of conceiving the thing?

If the harmony of intellects affects us when it relates to the manner of conceiving exterior things, it cannot leave us indifferent when it happens with reference to the ends we ought to pursue, the aspirations and ambitions of heart and will. Indeed, the instinct—so strong within us—which makes us imitate our fellow-beings, is a sort of spontaneous expression of the infinite importance we attach to the union of minds, and the harmony of these latter, once it is realized, shows itself as an important proof of the excellence of the end pursued.

As a matter of fact, this harmony itself is something into which we must enquire. The sole fact that a crowd of men is moved by one common impulse does not prove that its will is a good one. The very seductiveness of discovering that there is unanimity leads us to confound a

unity consisting of blind impulse, passion and more or less irrational will, with that unity of profound thought and sound aspiration, which alone is valid.

What are the characteristics of that thought which, of itself, and without being verified by external reality, establishes solid harmony between minds? It neither is, nor can be, pure intuition. Man cannot apprehend an object, know what he is apprehending, and inform other men of the fact, without using some concept. For him, a pure intuition cannot be an object of knowledge: it is simply a mental state, the importance of which he cannot gauge. But again, on the other hand, the thought we are here endeavoring to define, cannot be a mere arrangement of facts. Concepts represent, on a small scale, acquired experience, and here we are dealing with ideas that transcend experience.

In reality, man has at his disposal a faculty called reason, the property of which is that it closely unites and blends together intuition and concept, so that the latter becomes supple, expands and enriches itself under the influence of the former, and this, in turn, under the influence of the other, becomes an object of consciousness, apt to be grasped and determined, understood and communicated by man. Reason is the close union of concept and intuition.

Mathematics, a rational science, if not in so far as it can be taught, at all events, in so far as it can be created, implies the close and constant collaboration of conceptual deduction with intuiton. It is by a judicious use of this living reason, the profound and original reaction of the entire intellect in contact with things, the soul of science and of life alike, that we can attempt to determine, not only facts or concepts, but ideas, types of what things may and must be if they would realize the utmost perfection of

which they are capable. According to this view it is advisable to attempt a definition of the essence of religion.

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The history of mankind is the richest and strangest, the most curious and varied spectacle imaginable. Customs, religions, institutions and beliefs, modes of life, aspirations, needs and ambitions, in all these respects the difference is incredible between beings to whom we attribute the same species. The turbulent life, too, of these beings has produced a variety of events disconcerting to the investigator; for the more the latter brings light to bear on the details of things, the more he sees complication, strangeness and novelty, where a superficial glance saw nothing but simplicity and uniformity.

Now, strange to say, every individual, every human group, not only keeps to its customs because they are its own, but because it regards them as superior to those of all others. Every human event, too, is not determined by an insatiable need of change alone; it ought, besides, in the minds of those involved in it, to realize a finer and loftier form of existence than all previous forms. No generation seriously imagines itself inferior to its predecessors. When we delight in analyzing the benefits enjoyed by previous generations and which we do not possess, the very eulogies we confer on our forerunners signify, at bottom, that it is the fault of none but ourselves—by appropriating what they have found useful and good—if we not merely equal, but do not also transcend them.

Now, what is the value of such judgments? Are they wholly absurd? Is human history, in reality, only a vain succession of uselessly diverse forms, or has it really a meaning? If we consider, not societies and their history, but the individual, in the consciousness he possesses of his interior life, we have the same problem. Man is constantly

seeking, with all his faculties, something different from, something better than what he has. A given sensation impels him on to follow after some new sensation. One idea presented to his mind suggests others, leading him to question, compare, philosophize. In his eyes, the goal attained by his will is no longer anything more than the starting-point of some new enterprise.

Whether in the history of his species, or in his individual life man is a being that aspires to transcend itself. What is the meaning, the value of this strange aspiration? Mainly by emphasizing certain aspects of modern science, in answering this question, it is possible to say that man is the victim of illusion, that nowhere in history could any phenomenon take place which is not simply an equivalent of preceding phenomena that gave it birth. *Eadem sunt omnia semper.* Wholly and from all eternity, the universe has been preformed, as regards its elements and laws. If man is conscious of something lacking, some possibility of growth, if he imagines that supernatural powers come to his assistance, these impressions are solely owing to his ignorance and vanity. His power is a given quantity, the mechanical resultant of the natural forces of which he is the accidental and temporay synthesis. His destiny is confined within the limits of this power.

This appreciation of things is very conceivable, and, in certain ways, it is a plausible one; but it is not necessary. Indeed, human reflection has always set forth quite a different interpretation, according to which, man has, in a very real sense, the power to conceive of ends superior to his natural forces; towards these ends it is possible for him to rise, because to his activity may be added that of some being greater than himself and more powerful than nature. In collaboration with this superior being, man may, in very truth, transcend both nature and himself. It appears as

though it would be advisable to seek for the essence of religion along these lines.

Man is on the path of religion as soon as he makes a serious endeavor to transcend himself not only quantitatively but also qualitatitvely. A purely quantitative increase of force might be explained by simply borrowing from the reservoir, infinite, it may be, of the physical energies of the universe. An increase of worth and perfection, however, if anything more than words, is greater than the forces of nature as such. Science and art actually aim at such an increase, but, depending upon nature and the given, they anticipate and seek after the true and the ideal, not knowing if they can attain unto it.

The originality of religion dwells in the fact that it proceeds not from power to duty, but from duty to power; that it advances resolutely, taking for granted that the problem is solved, and that it starts from God. *Ab actu ad posse,* such is its motto. "Be of good cheer," said Jesus to Pascal, "thou wouldst not seek me hadst thou not found me." God is being and principle, the overflowing spring of perfection and might. He who shares in the life of God can really transcend nature; he can create. Religion is creation, true, beautiful and beneficient, in God and by God.

Religion, ascending to the very source of being, interests the whole man. To enquire whether it is rather a matter of feeling, of intellect or of will, is useless. It has its abode in that inmost shrine of the soul where the one and the many interpenetrate; this is a characteristic which appears in what we call life. Here, will is faith, confidence, invincible determination, as becomes the man who feels himself one with creative power. Intellect endeavors to create forms capable of representing that which cannot be represented in a way that is both worthy of the object and comprehensible by mankind. And feeling, which is in turn

a sense of terror before the unfathomable and of enthusiasm before the divine, finds its full satisfaction in that supreme love—both possession and the gift of self—which constitutes fruitfulness and joy *par excellence*. All these manifestations, at bottom, control and permeate one another, just as, in the case of light, those colors are united which, if reflected from different bodies, would be distinct.

Is religion, however, only an interior form of the soul, and must we regard as purely accidental the phenomena by which it is manifested externally? Nothing could be more contrary to its essence than a radical distinction between spirit and letter. Since, above all else, it is participation in creative power, and therefore creation itself, religion naturally tends to set its stamp on things both visible and invisible, to express itself in forms, symbols and rites, and to mould the whole of human life. These visible expressions, the connecting link between the principle of being and undulating nature, necessarily vary with time and place. Their importance, like that of all translation, lies in the degree in which they reconcile fidelity to the original and conformity with the special conditions of the language employed.

The mode, too, in which religion acts, is essentially spiritual. It necessarily proceeds from within to without, not *vice versa*. Indeed, it constitutes the might of the spirit. Were it to appeal to force, it would be false to itself. Its mission is to permeate force, its triumph would consist in transmitting this latter into love. Its mode of action is summed up in the prayer: "Father, thy kingdom come, thy will be done on earth, as it is done in heaven!"

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DOES "EVOLUTION" EXPLAIN?

THE demand for a causal explanation of nature processes is inexpugnable from the mind. It is idle to tell us that one event or condition is habitually preceded or followed by another because they are thus juxtaposed in our experience—idle to suggest with Hume, Comte and their followers that all we need to know about change may be summed up by a classification of it into a series of regularly recurring successions. When the earth intervenes between sun and moon we want to know something more than that our satellite is darkened, something more than that when flame is applied to gunpowder the contact is regularly accompanied by a sudden evolution of gas, something more than that the presence of certain bacteria in the blood is the invariable concomitant of a particular disease. The habitual juxtapositions, the regularities of occurrence, the sequences of mode, must be observed and noted. But these do not suffice. The search for cause, however minimized or denied, is implicit in all scientific investigation: nature knowledge has been advanced and our power of predicting phenomena increased just to the extent that to knowledge of the uniformities of succession we have added a theory of the properties or powers which produce the succession and preserve the uniformities. Beneath the "how" of every vicissitude there is always a deeper "why." Things do not change spontaneously. We require a reason, a sufficing ground, an impulsion of some sort, capable

of satisfying us that the change must take place. If bodies are set in motion or brought to rest they are pushed or pulled: there is differential stress, with gravitative, electrical, chemical or muscular action at work as a determining force. We may not be able to "explain" the ultimate cause of change, but we must assume one, and our formal descriptions of the sufficing reason, expressed in terms of force, do really constitute explanation in the only sense in which we can use that term.

Now the doctrine of evolution is essentially a doctrine about changes. Opposing the old view of creation "all of a piece," whether out of nothing or out of something, and the equally untenable view of creation by sudden metamorphoses, it teaches that objects were not always as we see them to-day, but that they have assumed their present shapes and configurations by a process of gradual transformation, and that the vicissitude to which they are subject is still going on. Yet it is far more content to sum up the order of the changes than to offer an account of their producing determinations: it deals, in fact, with formal successions rather than with causal successions—tells us the how of vicissitude without feeling called upon to set forth its why. Fruitful as a formula, the doctrine has been successfully applied to the whole range of the phenomena accessible to us. But the very universality of the process—the ease with which the evidences of it can be accumulated, and the vivid effect which its concrete examples have on the mind—lead inevitably to an exaggerated interpretation of its nature and scope. Evolution is the mode in which material forms emerge or "unfold," yet it has been lifted to the rank of a primary power in the constitution of things. For the popular thought of the time we see it adopted as a key to the solution of all universe problems. Say only that something has been "evolved," and the statement is received as equivalent to an explanation. Even scientific men have

sometimes surrendered to the spell of this magic word, and if we would know how completely a merely model account of nature can be erected into a causal elucidation of nature it is only necessary to recall the recent words of an American astronomer: "Evolution is nothing more nor less than the mainspring of the universe. Grand in its very simplicity, it is the one fundamental fact to which all we know is ancillary. From its influence nothing can escape, for it has fashioned everything, from nebula to man." (*Bulletin of the Society of Arts*, 1909.)

Evolution begins in the inorganic with the appearance of matter—born out of some differentiation of the ether which sets free the play of gravitative forces. But science is undecided as to the nature of the process. Half a dozen hypotheses are in the field. The ether is continuous or it is granular; it is a sort of labile foam; its density is greater than that of steel; or perhaps there is no ether. The matter unit is a coagulation of ether; it is a knot in ether; it is ether in motion; it is corpuscular; it is electrical. A similar uncertainty prevails regarding many of the details of evolution. What, at the outset, is the relation of motion to the thing that moves? We still treat motion either as an ultimate, which does not need explanation through being uncaused, or as a produced derivative the cause of which remains obscure. If all ether energy is kinetic energy, and the thing which moves is refined away into electricity or some other form of motion, we are left with motions that move—a whole universe of them—as the source of the solidities we call matter, while matter is reduced to types of motion in motion. If ether energy is non-kinetic we still await an account of how motion arises and of the relation which must subsist, for the needed unitary view of them, between the energy which is not motion and that which is. If force can be exerted only against resistance, it would be interesting to know why physicists continue to

distinguish between motion against resistance, as on the earth's surface, and motion without resistance, as is alleged in the case of stellar and planetary motions. If there is anywhere in the universe a no-resistance to motion, what becomes of the thesis that action and reaction are equal, and how shall we distinguish between the conditions which initiate change and those which determine no-change? Is conservation of energy limited to the realm of motion, or does it include conservatism of mass?

The doctrine of evolution, with a formula that includes retrogression as well as progress, interests us most as a theory of advance, and it is here, if anywhere, that we have a right to be satisfactorily served. The whole realm of matter, inorganic and organic, has been ransacked to show that there is progress from states of disintegration to states of integration, from the homogeneous to the heterogeneous, from simplicity to complexity, from the undifferentiated to the differentiated, from the incoherent to the coherent. Yet the descriptions do not always fit the facts. If sometimes the contrasted processes and conditions exclude each other, at other times they are manifested in the same aggregate and the same area. Differentiation frequently appears as means to a larger kind of assimilation; the molecules of any particular type of matter, the units of any particular living tissue—all species, in fact, whether organic or inorganic—are fundamentally systems of likes. The changes which constitute social evolution may begin with differentiation, but the progress is finally from the heterogeneous to the homogeneous. In our modern human societies this aspect of advance dominates all others: everywhere we see peculiar institutional forms, social customs, ways of living, even fashions of dress, yielding to an assimilative process which, out of community character and national character, is making world character. As invention runs the industries of countries into like moulds, so commerce assimilates

peoples by promoting like needs and supplying like commodities. With the diffusion of dominant ideas in politics, science, art, literature, even religion, the planet is more and more becoming a community of the like-minded. And even language, which perfects itself by discarding the analytic distinctions and cumbrous inflections of its earlier period, thus progressing not from the simple to the complex, but from the complex to the simple, is already preparing to throw off those isolating distinctions which still make the peoples unintelligible to each other. Throughout international life we thus find writ large the process which is constantly going on within the species, the group, the community, the nation—the setting up of those closer relations which lead to assimilation, the likening which is required for coöperation, and the coöperation which in turn gives rise to the larger forms of resemblance and homogeneity.

It seems to be assumed that somehow or other changes are caused, for without determination of some sort matter would be indifferently at rest or in motion. But evolutionary doctrine leaves great gaps in the connections. Its cogs are often out of gear, and its piston has no visible connection with the driving wheel, plain as it is that the machine works. A tremendous breach of continuity is noticeable in its treatment of the change from inorganic to organic. How progress is made from an object which moves only as it is pushed or pulled to a self-moving system which renews itself after waste and reproduces its like remains after centuries of investigation as insoluble a problem for science as that of gravitation itself. Now the organism must arise out of the inorganic as an expansion of characters regnant also there, if in a more elementary form: it cannot have been thrust into the world as a new kind of object totally unrelated in its powers and functions to objects in general. Yet the appeals to chemistry and physics, to vitalism and neo-vitalism, to psychism and panpsychism, enforced by

researches however praiseworthy, have all failed to explain the transition. We cannot be put off with verbal formulæ which merely re-describe the problem they pretend to solve. It is not enough to tell us that organic matter is highly complex, possesses irritability, is capable of reacting to stimuli, assimilating nutriment, and the like. We want to know how these properties and powers emerge from the inorganic. We need to have cleared up for us—in all its connections, with all its pushes and pulls—the something which makes the living unit not merely dependent in the dynamic sense, but interdependent in the organic and structural sense; which sets up the demand by each for all, the insistence by all on each; which works only through the subordination of part to whole and the domination of part by whole; which, in a word, turns a congeries of molecules primordially indifferent to each other into a society of units system-sourced rather than self-sourced, to impel them one and all towards the self-maintaining activities and configurations that we call life. The analogies which thus relate the individual organism to the species of organisms and finally to human society as a whole must be capable of extension far beyond the limited realm of the living to the sources from which both animate and inanimate derive. But biology is too much concerned with the minute to seek the solvent for its problem in the large. The effect to explain vital characters spends itself mainly in exploration of the microscopic detail of cell structure. Study of the behaviour of the egg-cell after fertilization deals with a secondary process arising out of sex; and Mendelism, frequently mistaken for an explanation of heredity, is no more, if no less, than an account of one of its complications. Supremely untouched by any of these modern conquests of biology is the power of living protoplasm to reproduce its like. Solvents of secondary vital phenomena multiply, but life, out of which they proceed,

remains unanalysed and mysterious. In the attempt to elucidate the organism from itself science misses that large grasp of the nature of life which connects its creative function with the realm of the inorganic and finally with the cosmos.

As the evolutionary view fails to relate vital to prevital characters, so also does it fail to define the inorganic characters out of which heredity emerges as by necessity. The "dominants" of Reinke, the "determinants" of Weismann, the "engrams" of Semon, have all been called in to explain heredity, but their elucidations have been only verbal: they apply to cases in which heredity has become complex, or they subsume a physiological memory inseparably implicated with consciousness. Most scientific discussion in this field takes heredity for granted, as most scientific discussion regarding the "origin of species" takes life for granted. In both the interest centres in secondary results and in complications of these. Given germplasm, what is its relation to the soma or body of the organism? How do the fertilized germ-cells divide, the chromosomes arrange themselves and the processes of ontogenetic development begin? Why is asexual reproduction followed by sexual? Is amphimixis an accident, a mere form of nutrition, or a means of providing material for variation? Do body changes pass to the germ-cells, or do these vary on their own account? Are variations minute and gradual, or "kaleidoscopic?" What part does natural selection play in organic evolution: is it neo-Darwinism or neo-Lamarckism to which we must pin our faith? In all these matters science is at work, not in the foundations of vital phenomena, but on their superstructure. What we need supremely to know, as the very foundation of all other knowledge about the organism, is why and how matter, beginning as inorganic, develops vital characters, setting up that new order in the world of things which we call life.

Yet for science the secret of this portentous advance has thus far been withheld.

Vitalism, whether new or old, offers no help through this *impasse,* nor is the way out shown by the new school of science which teaches that inorganic matter may itself be alive, if with a lower order of vitality—that it may even have a dim sort of consciousness which needs only complexity to make it "intelligent." Psychism in matter tends to panpsychism, and psychism read into the universe process as a means of explaining it stultifies the only kind of "intelligence" we know anything about. It is our habit —imposed on us by our own natures as organisms—to view all appearances of contrivance organically. As our own intelligence is conscious, all other forms of intelligence must be conscious: wherever matter shows adaptation to ends, there we call in consciousness to function as the directing power. We do this in defiance of the plainest evidence, for growth is completed and intelligent processes go on within our bodies without the slightest direct let or hindrance from our conscious states, these being concerned, not with the building up of the organism or with its inner march of events, but with the general activities and mental processes which are needed for relating us to the environment. What we need to have unriddled for us is that power which enables the organism to act intelligently, without aid from consciousness, by virtue of its very nature as living matter—that order which connects the limited organic product with its source, the organism with the inorganic, the animate system with the pre-vital cosmos. The world spreads out before us as if shot through and through with mind. How are we to explain the bond of common process which everywhere links the purposive adaptations of nature with the consciously wrought contrivances of man? Why should the universe, like the intellect, segregate and classify, bringing likes together and

separating unlikes? What is it that matches the rounded boulder with the sphered planet; that relates the spiral of the nebula and the shell to the volute of the Ionic capital, and these to the curl of the wave as it breaks on the shore; that passes from regularly spaced ether pulses to regularly spaced air waves, sand waves and ocean rollers, to reappear in the iterated lines and patterns of man's decorative art; that makes one the geometry of the crystal and the geometry of the honeycomb, the intelligence that shapes the dandelion seed to the wind and the intelligence which builds up human industry, navigates the seas and explores the heavens? Of all this evolution tells us nothing.

In another way evolution is inadequate. Seen from the natural side its rationality is unimpeachable. It is the unfolding of power that eternally is into processes and configurations, into motions, structures, shapes. But the moment we try to make its intelligence conform to that of man our difficulties begin. For human intelligence—the only kind of intelligence we know—a plan is formulated complete, and the fashioning follows the plan. There may be discardings and rebuildings as knowledge accumulates and as new natural properties and methods of utilizing them come to be discovered. But the relation between the mind and its end at any instant is immediate, not provisional or advancing. Now while the machine is a system which works only after it is made, the organism is a machine that works while it is in the making. For the human architect or builder a palace comes out in one piece, not as a series of progressive modifications from an original cave dwelling or mud hut. What engineer planning a bridge would begin with a rope, substitute wood, then put in iron, and finally replace the whole with steel, if in knowledge and resources he were adequately equipped for his task from the beginning? The "Learn to labor and to wait" needs modification even as a bit of worldly wisdom: to address it to omni-

potence is neither logical nor reverent. Yet it is precisely this method of thought and afterthought, of completion and revision, of overturn and re-making, to which the evolutionary teleologists refer the whole furnishing of earth and sky. From non-nucleated protoplasm to nucleated, from unicellular to multicellular, from pre-gastrula to ectoderm and entoderm, from invertebrate to vertebrate, from quadruped to biped, with accompanying changes in modes of reproduction and endless blind alleys of structure and type, nature is one long march from the provisional to the incomplete, one long powerlessness to reach its goal by achieving in a stroke what is needed at once and *de toutes pièces.* And as man finds himself fastened to the evolutionary method, not by the choice of his intellect, but by the nature of his body and its relation to the cosmos, so man is compelled with only rare exceptions that prove the rule to work out his own welfare by the same alternations of rejection and advance, the same lingering in stages that satisfy until they are outgrown, the same beckoning of goals that promise finality only to recede or vanish when they are at last won. It was evolution that armed our predecessor with a flint when he needed a knife, that imposed a clepsydra on him when he deserved a clock, that put him off with sails when steam power should have been his, that kept the stylus in his hand when the printing press should have been recording and perpetuating his thoughts. Perhaps the acquirement of all these good things had to be gradual? How gradually do missionaries introduce clothes-wearing, modern implements and other innovations to the savages they teach? If centuries of evolution are needed for animals and peoples to become up-to-date, how are we to explain the sudden transformation in countries like China and Japan? But "the time was not ripe," "the world was not ready." Would the millions who died needlessly from disease have admitted their unreadiness for antiseptics and

antitoxines in the days before sanitary science and preventive medicine? Were industrial operations ripe for the helping hand of the state only with the advent of factory inspection and shorter hours, women ready to benefit from humane legislation only when the order came to unharness them from the coal truck and the embruting labor of the mines? What was it that so long remanded the reform of asylum and prison abuses if not the necessity of waiting on evolution? Was serfdom right even in Roman days merely because it had not yet been denounced? Is war justifiable because its abolition has not yet come within the range of "practical politics"? That which we see in all these human delays is the what is dominating the what ought to be, the deferred right turning present attainment into relative wrong, the deferred justice making actual conditions relatively unjust. And to the retrospect, individual or social, the sense of deprivation can never be wanting. Nobody will ever accept the view that he has lost nothing by having meted out to him to-day that which ought to have been his yesterday.

The evolutionary method in knowledge, as psychically interpreted, is open to similar objections. Nothing can well be advanced against gradual acquirement and constant revision as a process required by the order of nature. But the claim that it has been imposed upon man by an intellect of like kind with his, to whatever extent wiser and more resourceful, arouses a demurrer from which no system of apologetics will ever free us. "Truth in the making" by all means if, in struggle, suffering, tragedy, we must make it for ourselves; but if we are to be led to it through endless doublings and zigzags by a guide who, knowing the straight path, refuses to reveal it to us, the temper of revolt must be ours along every inch of the hard and stony way. Knowledge slowly and adventurously won, satisfaction following curiosity, the strain of quest rewarded with the joy

of conquest, a thousand times welcome! But offer it as a gift consciously bestowed on the instalment plan—grudgingly parcelled out in bits, like gulps of nourishment administered to the starving who should have been earlier fed—and the intellect rebels. What men need at all times is not the error tempered by fact, the wrong view to be condoned because it is workable, the interpretation *faute de mieux*, the theory sure to be outgrown, but "the truth, the whole truth, and nothing but the truth." We do not instruct our youth that the sun goes round the earth, or that our planet is flat, with the remark that, when their minds are ripe for something better, such statements will be supplemented, revised or wholly superseded. What would be thought of a text-book in any branch of knowledge of which each chapter after the first should qualify, refashion or completely reverse the teaching of that which preceded it, or of which the closing chapter should form the introduction to a new series of affirmations and denials? The earth is the centre of the universe, and all bodies are subservient to it; the universe has no centre, and our planetary habitation is a moving mote in the star-spangled cosmos. The energies are distinct, gravitative, chemical, electrical, vital; the energies are varieties of one all-inclusive energy. The atom is the indivisible ultimate; the atom is divisible, and the electrons are the ultimates, or the ether unit is the ultimate. Mass is invariable; mass is variable according to speed. There is matter and there is electricity; there is no matter, and everything is electricity. Gravitation takes time to "travel;" gravitation is instantaneous. Heat is a substance; heat is a mode of motion. Light is an emanation from the eye; it is an emanation from objects; it is corpuscular; it is made up of pulses propagated in ether; it is distinct from electricity; it is an electromagnetic phenomenon. Life is divinely originated; it is due to a vital principle; it arises out of the physico-

chemical properties of matter. Life is created and species are immutable; life is evolved and species are mutable. The germ-cells are produced by the organisms in which they appear; they are left-overs from previous generations. Consciousness is the driving power of life; consciousness is an epiphenomenon or a psycho-physical parallel. Mind is the summated result of separate items of consciousness; mind is the unity which includes and dominates all these items. And so through the whole round of the sciences, from physics to psychology and from astronomy to sociology, we confront the same resting in half knowledge until it is outgrown, the same progress from old error to new error, the same advance from knowledge invalidated to knowledge provisional, the same lure towards finality and the same disappointments. The method of deferred truth is thus no more satisfying than the method of deferred justice. We accept them as the outcome of a nature of things with which consciousness, will, personality, can have had nothing whatever to do. Viewed as the result of plan, they stultify the only conscious intelligence which is known to us and the only mind of which we can rationally conceive. But give us the universe for our support, with a relative freedom to work out justice and achieve truth for ourselves, and the cruel antinomy dissolves before our eyes.

Any survey of this kind is obviously fatal to the psychic interpretation of nature. In the deepest aspect of such a view we find it weighted with the determination to see the universe not as all-productive and multi-potential, but as restricted to the development of mass and motion, of inter-determining forces, of bodies spatially limited and subject to causally produced change, of inorganic objects maintained and of organisms self-maintained, of life requiring knowledge for its safety and furtherance. And within this aspect, concerning us more nearly and on that account all the more misleading, is our tendency to assimilate the

cosmos yet further to ourselves by reading into it the characters of vitality, will, consciousness and mind—by linking it to human interests and conceiving of our relation to it in terms of human values and human aspirations. Now whatever this attitude may do for religion we cannot regard it as advancing science. The ambition of science, however disguised, is to explain, and the psychic interpretation does not explain: alike in return to the primitive ensoulment of nature and in resort to a supposed elementary vitality in the atoms, it merely adjourns a problem which the very hypothesis admits to be incapable of solution. We might as well try to account for the flower by assuming that stalk and roots are made up of units each of which has something of the flower character, or for water by assuming that it must be composed of atoms and molecules each of which has the water "property." We have here in physics, as once before in biology, the hypothesis of preformation where a theory of epigenesis is needed. Neither vitality nor consciousness can be traced to really ultimate units: they imply and require collectivity. And if we expand the doctrine into panpsychism we confront the insuperable difficulty of realizing how psychic characters ancillary to self-maintenance can either arise or function in an inorganic realm which shows neither activities nor structures that can rightly be called organic. Even if consciousness could be envisaged as a kind of universal entity—vaporous, ghost-like, or what-not—ready everywhere to invade matter, get a grip on it, and carry it along, as Bergson suggests, "to organization" —we should still be left with our puzzle all unriddled; the gap between organic and inorganic would continue to yawn, or would be bridged, not by causal relations, but by formal propositions; the appearances of plan in nature, the suggestions of design in plant and animal, the plain evidences of intelligent adaptations unconsciously reached throughout the realm of life, would receive a merely verbal,

not a real explanation. As by no assumption of an *élan de vie* do we approach even a pretence of accounting for life, so by no assumption of psychical "jets," "spurts," "leaps," "impulses," "upspringings," arising anywhen from anywhere, can we supply the links that are needed to relate the organism to the cosmos.

In seeking extra-organic aid from the *psyche,* whether scaled down to the "electrons" or scaled up to the universe, science is taking a road which—to the defeat of the very method on which its triumphs rest—can only end in a wild subjectivism. It has already accumulated knowledge enough for a provisional view of nature; all that it now needs is a rational use of the materials accumulated. And the goal to be reached clearly lies in the direction of realizing that the universe is infinitely more for itself than it can ever be for man. We must first give up our fatal habit of assuming that the cosmos can be lifted into respectability, as it were, only by projecting into it the local and peculiar characters which we find so valuable to ourselves. Subjectivism in the past has exalted us to a proud superiority over the inorganic things which it has been convenient to sum up as the ether, as matter, as mass and as motion. We have disparaged the world of power about us largely because, impressed by the visible and tangible, we early fell into the mistake of using them as a label for the universe in its totality. A cosmos with a vaster and richer content, alike of actuality and possibility now awaits our contemplation; and within its unutterable sweep we are to discern a system which becomes intelligible to us, not through our ability to spiritualize it, but through our power to interpret the phenomena it presents in the light, so far as may be, of its own nature, process and modes. The time has come to trace to power itself—to the accessible ground of the energies, forces, bodies, motions which we call cosmos—those appearances of design which, in the psychical interpretation

of them, have been subject to age-long acceptance as indications of "mind in nature." But to do this we must emancipate intelligence from necessary implication with consciousness, just as we must emancipate power from necessary implication with motion.

What do we mean, what ought we to mean, by "intelligence?" The subjective element of the conception is most emphasized when the biological sciences are not in question. Yet it is possible to think consciousness wholly away from vital activities without disturbing their modal character as end-reaching, while it is impossible to abstract mentally that character and conceive of them as "intelligent." Even when thrust most vividly upon our attention, consciousness appears only as an added instrumentality, not as a constitutive and fundamental principle. To realize how little we are entitled to look for it in the inorganic world we have only to remember that consciousness is not needed for the vital processes that go on within our own bodies and shades off to a vanishing point in plant life—only to recall that the living body has fashioned its organs unconsciously and without anything of the nature of plan aforethought. Why, then, in the lifeless, the inorganic, should we find "intelligent adaptations? The answer can only be that intelligence is not primarily and fundamentally conscious, but fundamentally and primarily dynamic; that the process we have so long tried to interpret psychically is rooted in the very modes of motion to which power gives rise; that we are in fact dealing with a universe which, strange as the paradox may seem, is "intelligent" before it can be conscious, and purposive before it can develop the faculties of mind. Accept this deliverance from the psychism which, after capturing metaphysics, now threatens science, and our most serious difficulties regarding organic evolution disappear. The self-maintenance we call life links up with the more elementary form of self-maintenance which we know as the

conservation of mass and the conservation of energy. Intelligent action by the organism in relation to environment, shown especially in the invention, fabrication and utilization of tools and machines, connects just as naturally with the unconscious artifice by which the living body develops organs within itself; so both relate themselves to their source in that vaster cosmic intelligence whose ordered configurations in the pre-vital realm appealed to the older teleologists as "evidences of design." For like reasons the account of organic progress as due to a "survival of the fittest" subordinates itself to an account of the origin of the fittest; chance in both organic and inorganic gives way to direction, the purposive is seen everywhere dominating the mechanical, and the real evolution comes into view.

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THE GENERAL NATURE OF THE CONDITIONS WHICH DETERMINE DEVELOPMENT.

THE saying of the witty *Autocrat* "that the brain often runs away with the heart's best blood" has its application to Philosophy, for every great system of thought springs from profound feeling—from an intense passion for order, a deep longing for unity. In the philosophy of science of the last few decades this was expressed in the conception of sociological phenomena as being organic, or even (at a somewhat earlier period) as in some important aspects physical;[1] and more recently in the comparison drawn between the electronic atom and planetary groups. But the inadequacy of the conception of society as an organism has long been manifest; so that to-day some universal term is required that shall be truly all inclusive, and capable of expressing the concrete character (should there be one) of all reality.

But any such universal character, if it were also truly concrete, would be scientific as much as philosophic—it would be philosophical as expressive of *all* reality, and scientific as applicable to *every* reality. Two modern thinkers agree, however, though for widely different reasons, in rejecting any such identification of science with philosophy;—Bergson, because science is vitiated by the defects inherent in the intellect, and therefore cannot apprehend

[1] *Cf.* Bagehot's *Physics and Politics.* Plato had discovered much in common between the spirit of the individual and of the community.

real duration; and Croce, because science is essentially non-philosophical, and employs not the true concept but always pseudo-concepts.[2] On the other hand Hegel endorses this principle, at least formally; for his first category—Pure Being—although too abstract to permit the thought of "every reality," is still, while purely abstractly, expressive of "all reality"; on the other hand the supreme category—Idea or Spirit[3]—would prove extremely difficult to apply directly in any sense consonant with the present usages of science. But if we follow Dr. McTaggart in his contention that the Hegelian Idea implies that "The Universe is differentiated. It consists of an organic system of individuals," we obtain at once a principle[4] fundamental to all science both in its most general and in its most specialized aspects. The Universe is, in the first place, differentiated; as such it is a system (not for science but) for philosophy; and further, it is differentiated throughout —it is a system of individuals, each of which again is itself a subsystem, great or small as the case may be, and each, from this point of view, constituting the subject matter of some department of science.[5]

If then the term "system" is given its widest and most profound meaning, it becomes equally applicable to all reality and to every reality, and thus the concrete universal of philosophy finds its proper place also in the realm of science. It is certainly unfortunate that the usual meaning of "system" is narrow and specialized; we think of "business" or "memory systems" etc.; still even in these the really essential feature is present in the sense of the completest possible organization—the absence of all super-

[2] *Logic as the Science of the Pure Concept*, p. 46.

[3] *Cf.* McTaggart, *Commentary*, sec. 294.

[4] *Loc. cit.* sec. 292; mathematical procedure is analogous in the transformation or degeneration of equations.

[5] Or other department of knowledge which cannot however, in view of the unity of all knowledge, be severed from science.

fluity and irrelevance. Instead then of regarding society e. g., as an organism, it is philosophically truer to look upon both alike as systems which, while differing in their special details, are similar in their possession of coherence, relevance and interconnection.

The universe again (still following here Dr. McTaggart's reading of Hegel) is an *organic* system; just as no organism is static, so is every natural system essentially dynamic—changing, evolving, enduring. Certainly we may, but only by limitation and abstraction, distinguish states of static equilibrim of shorter or longer duration; but this static aspect is always merely superficial or (in a sense) illusory and, always underlain by force or activity, is but a temporary though necessary stage in the total development; all states of equilibrium arise out of dynamic conditions which at the moment counteract each other. If then we assign to "system" the dynamic character which is distinctive of all living organisms, we are enabled (I think) to trace connections which unite or (to coin a word) "monify" categories of existence apparently widely separated, even if the actual detailed evidence for this is very slight. In this way "vitalism" and "mechanism" become complementary to each other instead of being antagonistic;[6] and I have previously endeavoured to prove that the development of the universe is necessarily a continous advance and never, as a whole, retrogressive.[7]

At first sight, certainly, the concrete phenomena and conditions of this development are so infinitely diverse that knowledge seems compelled to remain always departmental; each science having its own distinct principles, applicable within its special sphere, but yielding at best only analogies, perhaps faint and distant, to other realms of

[6] I may refer to *Science Progress*, No. 50. p. 305.

[7] *The Monist*, April 1920, p. 203. Perhaps "development within the Universe" would be a better expression. How a Universe can truly develop is of course an extremely difficult problem.

truth; thus the laws of motion are as foreign to psychology as are the principles of association to chemistry. Within limits, and mainly in the abstract sciences, this "irrelevance of determining principles," as it may be called, is slowly vanishing; thus mass and energy would appear to be identical, and all forces are probably electro-magnetic.[8]

How far then is it possible to carry this principle of the monistic transcendence of externality? To what extent is it possible to determine *perfectly general* conditions of development?—to express *i.e.* conditions which govern every type of change whether simple or complex, physical, vital, or social. Can we ascertain the universal characteristics of (*a*) all embryonic or early stages, (*b*) all mature and stable stages, whatever be the detailed course development may follow?

There has first to be considered the concept of "origin." Today it is almost a platitude to say that everything has, and indeed must have, an origin; but the obviousness of this assertion should not obscure two important points—(*a*) that the problems of origin are always amongst the most difficult to solve and (*b*) that earlier stages of knowledge, either explicitly or implicitly, assumed that these problems were actually insoluble, and fell back upon statements of which the opening words of Genesis may be taken as the type.[9] Even today it remains true that no assigned origin is ever absolute, and that some degree of relativity, of arbitrariness, or even of conventionality, determines which stage or set of conditions is regarded as original. Subject to these considerations I should like to consider how far the

[8] The irrelevance is at its minimum when knowledge is most abstract *i. e.* in pure mathematics, which may be regarded as an extensive and increasing application of a few simple principles, applicable also, directly or analogically, within formal or symbolic logic.

[9] This does not imply, however, that such utterances have wholly lost their value; at the same time this necessarily changes relevantly to the growth of knowledge as a whole.

universal or perfectly general attributes of "origin" can be ascertained.

Every origin, in the first place, however simple it may be, must possess some degree of systematic structure, which again must be, either patently or covertly, of such a nature that some measure of continuity (or in more philosophic language, identity),[10] exists between it and all later stages. Here again we are often driven back upon more or less arbitrary principles in order to establish such continuity. How far *e.g.* is it true to say, in view of the immigrant flood, that the national characteristics of the United States are identical with those of the thirteen colonies? and the same problem appears in a different form if we ask how the ingestion of food and the resultant tissue renewal bear on the identity of the organism.

But wherever there arises this difficulty of determining, from among any series of connected stages, the actual origin, it plainly indicates the absence of any absolutely distinctive criterion; it implies (in other words) that all the stages, whatever their degree of difference may become, are still alike in the possession of some common quality. This universally common quality is (once again) systematic structure; development, that is, in all cases and at every stage consists in the combination[11] of different constituent systems among which, as systems, it is frequently very difficult or indeed impossible to determine the essential origin and continuity; and it is only when one of the many systematic structures becomes in itself obviously dominant that it is selected as the real origin, or basis of true continuity. But this dominance or obviousness is always plainly a matter of degree and of relativity; the immigrants *e.g.* during

[10] Not however the bare identity A=A of formal logic. It is obvious that till this abstract self-identity is somehow transcended no knowledge is at all possible.

[11] In exceptional cases, dissociation, which is merged however in some later combination.

any selected brief period are much more loosely connected than are the United States of that same period; on the other hand, if the German invasion of Belgium had succeeded, the Belgian nationality would have been so radically altered that history would probably have been compelled to recognise a new "origin," as it has often done in times when armed conquests were the rule rather than the exception.

For a philosophy or knowledge ideally complete then every such contributory system would be equally essential, because such a final philosophy would possess the Absolute as its criterion of universal relativity;[12] our actual knowledge, however, must in its incompleteness adopt now one criterion and now another; thus, to cite one example, in the evolution of humanity "origin" and "continuity" assume wholly different aspects according to whether we view personal development as an end in itself, or as contributory to the existence of nationality or humanity as a whole.

But further, in thus insisting on the necessity of the conception of systematic structure, it is obvious that we have passed beyond the consideration of mere change as such. The idea of change merely in itself is superficial and abstract, and of little value beyond very limited ranges of phenomena.[13] Change must be regarded as occurring always within a systematic universe, and as itself therefore sharing in and contributing to its systematic nature; thus viewed it becomes the abstract aspect of some concrete development, and denotes in reality the progressive combination of diverse systems into an increasingly complex whole.

[12] It seems too often forgotten that "relativity" is itself relative; therefore not being itself absolute, it necessarily implies an absolue; as *e. g.* Time in Newtonian mechanics, and "proper time" in relativity mechanics.

[13] But the concept of change has always held a prominent place in philosophy from the πάντα ῥεῖ of Heracleitus to our own day. *Cf. e.g.* "What I find when I look at consciousness is a sequence of different feelings." (Shadworth Hodgson, *Philosophy of Reflection*, vol. I. p .248.) "Such a description", comments James, *Principles of Psychology*, Vol. I, p. 230), "can awaken no possible protest."

I think it is important to notice that this is not a conclusion which, at the present level of knowledge, can be obtained on mere *a priori* grounds; even if there are (as I believe) sound general philosophic reasons in its favour, still a "kaleidoscopic" universe—one *i.e.* in which perfectly haphazard changes constantly occurred which, though governed by law, still served no final end—does not seem *prima facie* to be wholly inconceivable; and in what sense such a world could be regarded as truly systematic is a difficult question.

Actual experience, however, presents to us development from origins; thus far we have seen that in all cases where it is difficult to discover the true origin, this difficulty resolves itself into that of the lack of criteria which enable us to select from among the many cooperating systems any one that is indubitably basal and dominant. But this conclusion has an obverse aspect which I should now like to consider—the problem *i.e.* of those attributes on which our choice of origin is actually based in cases where it proves to be quite well founded.

One such attribute that is very often insisted upon is the simplicity of origins; but this merely in itself is plainly an insufficient, and may even become a wholly misleading, criterion; and this for two reasons—(*a*) because simplicity is itself always relative, and (*b*) because *every* stage in development is in some aspects simpler than succeeding stages. It is true that we can often trace this increasing simplicity back to a point where a single further step would bring us to an earlier system which itself appears highly complex, and we then feel justified in placing the line of distinction between such a "parent" system and the "simple" system which springs from it.[14] But however legitimate

[14] The living germ is the most obvious instance; but this should, for conscious beings, include mental phenomena, which vastly increases the difficulties. But almost all social origins are on the same footing as regards their simplicity.

this procedure may be, it raises two obvious difficulties—(*a*) as to the nature of the "simplicity" of the origin and (*b*) as to its relation to its own previous origin or parent system.

In the first place, it seems exceedingly doubtful whether the "simplicity" thus attributed to origins has any actual existence; here we may often be completely deceived by appearance. Certainly all investigation shows that the living germ is exceedingly complex—as complex indeed in one way as is the adult organism in another; and this again is confirmed by the degree in which the germ itself largely determines the total course of development, which then comes to resemble the breaking down or the unfolding of a pre-existing highly complex but much condensed structure; an interpretation which is still more obvious in the case of the radioactive elements. Here then it is not so much a question of different *degrees* of complexity as of different *modes*, and of the gradual conversion of one mode into the other. Expressed again in still more general terms, all germinal stages appear to have a higher degree of potentiality than the later, so that in this respect the usual contrast as to simplicity is actually reversed. If we compare *e.g.* the spiritual endowments of the "simple" early Roman republic with those of the 'complex" but decadent Empire, there can be no doubt as to which possessed the greatest potential capacity; in all such cases the child is, in a very real sense, the father of the man. And whatever were the defects of the earlier "preformation" or *emboitement* theories of vital development they at least recognized and expressed (if only by implication) the high complexity of all the various stages.

Thus the essential character of origins is not so much the degree, as the mode, of their complexity. Their systematic structure is such that each of the constituents, while

all remaining united together, is capable of entering into a series of combinations continuously, until a new system has been constructed whose elements are so combined *inter se* that few new combinations remain possible. It is obvious that in both cases alike the *degree* of complexity must be very high; the only difference then must be between the two modes; for only a highly complex system will be capable of providing the innumerable bases and connections whereby the resultant system can arise; only an exceedingly complex structure can be thus active and potential, and the simplicity so often ascribed to origins is very largely illusory and abstract.

Development then—*i.e.* the change from original systems to adult—is best interpreted not as an increase in, but as a continuous transformation of, the mode of the complexity of the developing system; and it is obvious that if this transformation were to become complete, development would thereupon cease. In individual cases it does thus come to an end; and the transcendence of this finality is rendered possible only by the conservation, in some way or other, of that special mode of complex structure characteristic of original systems. The most concrete instance is that of the reproductive elements in living organisms; but analogous phenomena occur wherever we find expansion and development—in the spiritual faculties of creative genius—in the adventurous and enterprising members of a nation, be that primitive and obscure or mighty and famous. Thus the maintenance of universal development depends on the union, in all the component individual systems, of two radically different modes of complex structure—the one stable, more or less rigid, narrowly specialized in some limited direction, and therefore incapable of functioning in any other way; and from this arise the elements of regularity and mechanicality;—the other able to enter into a

long series of widely diverse combinations between which it functions as the common basis of union, and so contributes to the universe the contrasted characters of infinite plasticity and originality,—*l'élan vital*.

Reality, then, regarded as a developing whole, everywhere presents these two widely differing aspects; but it is fundamentally important to note that each is indispensable to the other, without whose activity it could not itself exist. This universality of mechanism in the world has always been a serious stumbling block to every philosophy that has considered itself idealistic; its apparent want of that freedom supposed to be inherent in mind or spirit has made it difficult to explain, while its patency makes it equally impossible to explain it away. On the other hand it has been as fully misinterpreted in the opposite direction by those crude materialisms which have taken it to prove that all spiritual values are only epiphenomenal; even Lotze, who, despite his wide scientific knowledge was no materialist, seeks its justification in that direct appeal to "divine wisdom,"[15] which is always a confession of philosophic bankruptcy. Bergson indeed does in a sense attempt to explain it away by regarding the mechanical as but the distortion, due to the intellect, of the true character of reality; and the basal defect of his system consists in this exaggerated insistence on the 'durational" aspect of the world and the consequent unjustified depreciation of all mechanism. It is necessary, of course, to draw between them a profound distinction; and it is probably inevitable that every philosophic system must somewhere place too marked an emphasis; but Bergson's dichotomy is altogether too absolute. For he regards two elements which are in truth complementary and so equally real, the one as merely illusory, while only its fellow

[15] "Conceiving it not as a self supporting fate but merely as a product of divine wisdom;" *Microcosmus*, Vol. I, p. 399.

is truly real; and two correlatively different methods of apprehension again—the one intellective, the other intuitional—then subserve our full consciousness of real being. But such an attitude wholly misinterprets the status of those fixed and completed elements in the world which are the fruits of its eternal process; between these and the processes which brought them into being there need be no antagonism whatever, provided they themselves conserve and maintain, as they obviously do, the means necessary for further development; each then—process and product, the mechanical and the dynamic—is the indispensable basis and complement of the other; nor does any impassable barrier prevent the mind from apprehending the real nature of either; for thought, even when farthest removed from full reality, is a self-condemned exile, and finds, as genius ever does, new life and strength in the deserts of abstraction, whence it returns armed with weapons still more powerful for its conquest of the concrete.

There still remains to be considered the question: In what consists the specific potentiality and complexity of all origins as such? What enables them to initiate and control new courses of development?—new *i.e.* even as individual instances of some generic type, much more when there is no pre-existent class to which they must conform.

The essential characteristic appears to be the advanced level of organisation which all origins possess, relatively (*a*) to their parent systems and (*b*) to the environment in which they can best function. In some respects or other this plane of organisation is always higher than that of both the environment and the parent source; and therefore every element in such an original system[16] is capable of entering into combination with other systems which are

[16] Except those necessary to maintain the system's existence as an origin; *e.g.* the cell wall of organic germs.

(*ex hypothesi*) lower in the level of organisation than itself; for none of these constituents can find its proper complement, or can satisfy its powerful affinities, within the limits of the origin itself. The resultant systems again are (also *ex hypothesi*) incapable of repeating that precise form of combination from which they arose, so that the original capacity for combination becomes automatically more and more limited at every stage in the process; in other words the potentiality of combination necessarily falls as the stability or fixity of the resultant system rises. Potentiality becomes gradually converted into actuality—the dynamic into the static[17]—the "durational" into the mechanical—without however either of these two fundamental aspects acquiring a degree of reality superior to that of the other, as is contended (in one direction) by the philosophies of materialism or of mechanical determinism, and (in the other) by the intuitional philosophy of Bergson.

If we venture to carry the enquiry still a stage further, and to seek the ground of this higher level of organisation to which all origins attain, we can only fall back (I think) on some such general philosophic principle as I have endeavoured to establish in a previous number of *The Monist*;[18] but a few additional remarks may be helpful regarding the difference which exists between the possibilities of organic, and of sociological or historic development.

Regarding the phenomena from the widest possible standpoint, the plasticity of organic development may be called specific rather than individual; the limits of the group *i.e.* appears to be more elastic than those of the individual, whose development is in the main confined to a fixed course from which little deviation is possible. This seems to be true whatever importance mutation theories may prove to

[17] But see the previous remarks on the relation between these.
[18] *Cf.* note 7 *ante.*

possess; for unless a group mutates, the variation is almost inevitably swamped so that reversion to type is assured.

This appears to depend on two causes—(*a*) the fixed character of the organic germ, which again is a necessary result of its material nature; and (*b*) the comparative constancy of the environment of the organic world, which also arises from its material constitution. To what degree the germ plasm is alterable by external influences is still a vexed question, and the inheritance of acquired characters an open problem. The invariability of the environment, again, both as regards its organic and its inorganic factors, is largely relative; nevertheless the operation of these two causes taken together would appear to explain the rigidity (as it may almost be called) of the course of organic development in the individual.

Social and historic progress, however, is on an entirely different plane. Freed from the bonds which confine the world of matter, wholly new influences come into play whose range and capacity are infinitely wider than those which rule the organic;[19] and though History may repeat itself in principle, it can never do so in details. All spiritual development therefore, both national and individual, proceeds within an environment which is itself rapidly developing, and every evolving system becomes an active and powerful factor in modifying the growth of its fellows. It is difficult to say in this respect whether personal or group action is the more effective—whether, in other words, the "great man theory" of History is well or ill founded. The same influence is operative in both cases, but is more concentrated in one than in the other. In every community destined to achieve greatness there must exist (as we have seen already) groups and institutions capable of combining with others, it may be externally through colonisation or

[19] "The Genesis of Freedom," *International Journal of Ethics*, April, 1920.

territorial expansion. On such a community the action of a vigorous personality is comparable to that of an enzyme or catalyst; it controls the social groups which it finds ready to hand and amenable to its influence, having the advantages of more rapid responsiveness to opportunity and less inertia than the group. On the other hand the conservatism of the group acts like a fly wheel and ensures some measure of persistence and continuity.

A few final remarks on the conditions governing the material basis of organic germs may not be out of place. During the protracted fall which preceded the solidification of the earth the formation of chemical compounds would plainly depend on two factors—(*a*) the quantity of the combining elements and (*b*) the strength of their affinity; both of these, again, depend on the electronic structure of the atoms. Their quantity varies with the readiness with which the electrons finally unite to form any stable atom, and the affinity of the attraction between the various electronic systems thus formed. An increased knowledge of atomic structure may reveal laws which express some constant connection between the chemical activity of atoms and their quantitative distribution.[20]

The continuous temperature fall again would necessitate the postponement of those combinations, due to the weaker affinities, which are possible only at low temperatures and in which the attractions are comparatively so feeble that the resultant compounds are extremely sensitive to external influences. This raises the problem as to how the elements concerned in these could be maintained in combination, particularly in any complex compound; probably radiation and enzyme action are among the active factors.

[20] The great range of carbon compounds appears to be connected with the exceptionally high frequency of its atomic vibrations. Gases, again, lead to maximum molecular disorder.

One further important principle seems to be obvious—that in all such complex and sensitive systems the various elements can be present only in extremely small quantities, because only so could they all be brought effectively within the range of mutual action. Thus there is an order of quantity, acting as a limit applicable to all the elements, which cannot be transcended without destroying this sensitive complexity; plainly again, should any one element increase in quantity, each of the others must increase *pro rata* to maintain that particular combination as such. This would be one factor regulating the volume of the organic cell.

These principles may be further extended to complex structures other than purely chemical—to all complex systems which are in consequence highly sensitive to external stimuli; in all these the quantity of each constituent must be relatively small, otherwise (as *e.g.* in the brain) the total quantity requisite for complexity would become too unwieldy. Similarly the non-spatial character of ideal and spiritual attributes enables them to co-exist in their immense variety, so as again to ensure extremely sensitive complexity; and if any man frets because of his personal obscurity and transience, it may console him to realise that these are necessary conditions of all existence within a highly organised universe.

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"RELATIVITY."

A SEARCHLIGHT ON HUMAN PERCEPTION.

MODERN science and philosophy reveal with increasing emphasis that we superimpose our human qualities on external nature to such an extent that it has been seriously asked whether matter has any real or "absolute" existence outside of our mind. Many of us have sought comfort in the thought that we possess geometrical axioms and mechanical laws that were as untainted by human-sense perceptions as mathematics. Intellects on other inhabited worlds should find them as immutable as we. Albert Einstein has taken our last stronghold. Fortunately he is not only a destroyer but an upbuilder as well. Into dissent he brings harmony, if mechanics based on one hypothesis is more harmonious than when built upon many. Beyond an hypothetical world explanation we cannot hope to arrive.

We call our new hypothesis "The Theory of Relativity" because we can prove that no two observers can describe a physical happening in the same terms if they are moving relatively to each other; that their measurements of size, weight, velocity, direction of travel, of the object under observation will differ; and further that there exist no means of determining whose measurements are true or "real." Both sets of determinations are of equal value. The greater the relative motion of the two observers, the more will their measurements disagree.

This is very bewildering; is there then no reality, no standard, no "truth" upon which we can build our conception of the universe? Let us reassure ourselves; the relativist as much as the orthodox scientist deals with absolute quantities and it is our object presently to make their acquaintance, only his realities bear a similar relation to our perceptions as various frequencies of light waves bear to pigments. Our slowly through the ages developing senses are not yet able, and probably never shall be able, to grasp objects or occurrences in all their aspects or in their full extension; in particular they do not take sufficiently into account that *time* enters in a very intimate manner into all observations. We must try to grasp that all matter has extension in time as well as in space; that it is not sufficient to locate an object two squares ahead, three to the right, ten stories up, *i.e.* along three space axes, but we must also specify the point along our "time-axis" when the observation is made, if we are to correlate the object in question with anything else in the world. In the "absolute" world which we must picture to ourselves, "objects" must be replaced with "events" because when my eye measures the length, width and breadth of an object these measurements consist of a series of events radiating from the object and experienced by my eye in a string of happenings lined up in time and, similarly, any and everything that happens to, in and around the object takes time, and is strong out in time, so that its "real" existence cannot be described without adding time as a "fourth dimension." It is here our imagination balks, but it is just when we have formed an understanding of the statement that time enters into our world picture very nearly in the same manner as length, breadth and height that we also shall understand which measurements have absolute value, why the measurements of the two observers disagree, why gravity acts equally on lead and feathers, and

many other puzzles of science and, lastly, we shall obtain a glimpse of a universe which while without beginning or end, without bottom or top, yet is finite, but without boundaries.

Such a glimpse is worth our while; therefore let us struggle on with our time-dimension or fourth dimension.

As the difficulty we experience, when trying to picture time to ourselves as a fourth dimension, depends on the inadequate development of our senses, we must *intuitively* extend our vision and can most easily do so by first entering into the world of such beings who as yet lack the sense perception of one of our three space dimensions and examine the place time occupies in their universe.

Consider for instance the world of the dog. Our time does not exist to him. He has no concept of yesterday or tomorrow; he lives in a continuous now. *What then constitutes time to the dog?* His behavior indicates that while he has learned to *act as if* there were depth in the picture before him, he cannot conceive distance between two objects lying on one line ahead of him. Therefore when the dog runs along the road the surrounding world becomes a moving picture: the distant trees stand still and the fence posts rush them by. The post opposite his eye belongs to his present, the previous post to his past and the following post to his future. This is precisely the way our time comes in contact with our senses. Our third dimension therefore is *time* to the dog. The past fence post and the coming fence post belong as much to "history" in the dog's life as the past and the coming year are part of our history. The dog sees motion in the landscape before him where we see nothing but stationary objects because we conceive three instead of two dimensions. The dog may run in a straight line and he sees the objects near him in a uniform rectilinear motion; he runs around the corner of the house and the *house revolves*. These motions in the

time of the dog are natural to him; they do not evoke his apprehension. But let something move in our three dimensional sense and it becomes *alive* to the animal whether it be a motor car, a spinning wheel or the cat's own tail. Here is life, mystery and perhaps danger. The blind is an entirely useless adjunct to the duck hunt provided the hunter could keep motionless; strange forms do not frighten the animal, they only arouse its curiosity; but let the hunter move only a finger, sometimes only an eye, and life, that is possible danger, is there. If we now understand what time is to the dog and remember that it "runs across the face of the dog," that is *at right angle* to his space, then we may by analogy understand the real meaning of time in our own three dimensional world. Past, present and future represent motion in the fourth dimension, and this fourth dimension or time axis is at right angle to our space. Furthermore, "the past" to one dog is evidently "the future" to a dog running in the opposite direction, and similarly past and future is a relative conception in our time also and depending on our state of motion. It must be remembered, however, that there is an irrevocable past and an impenetrable future in our time which has no analogy in space. But physical measurements deal with time in a zone, analogous to the landscape in front of the dog, where past and future are purely relative and may even be reversed between two observers.

The table below gives a picture of the evolution of space and time perceptions parallel to the evolution of life.[1]

NO. OF DIMENSIONS	1	2	3	4	5
One dimensional sense-perceptions: Snail	Width	Motion in time	Life Mystery		
Two dimensional sense-perceptions: Dog	Width	Height	Motion in time	Life Mystery	
Three dimensional sense-perceptions: Man	Width	Height	Depth	Motion in time	Life Mystery

[1] P. D. Ouspensky's book *Tertium Organum.*

We must therefore admit that time shares in most of the qualities of space and that all objects, or more correctly, all events (for the substance of matter *is* motion) have extension in four dimensions.

A bridge between orthodox space and time on one hand, and the four-dimensional world on the other, is formed by the fact that light travels 186,000 miles per second and that this velocity can be determined "absolutely," *i.e.* without the uncertainty (apart from errors of observation) that impairs all measurements of smaller velocities, so that we can speak indiscriminately of seconds and miles in our four-dimensional world. To this new world we have given the name: *Space-time.*

Let us now consider why human observations differ from each other when projected into the components of space-time, *i.e.* when made in space and time separately. In our analogy from the world of the dog it is easy to see that his "time" changes direction, as the direction of travel of the dog changes, but for the more exact understanding of the phenonemon we may take an example from our three-dimensional world.

An observer on board a ship in harbor and an observer on shore agree as to the length, width and height of the ship. When, however, the ship steers out on the ocean the ship is gradually tilted with reference to the observer on shore due to the curvature of the earth so that the ship apparently loses in height even if our observer on shore always could see the whole ship. The line that is vertical to him forms a larger and larger angle with the vertical on board so that vertical is partly transformed into horizontal and vice-versa. Although vertical appears very distinct from horizontal to the observer on shore the identical direction has no specific meaning on board the boat at sea. Space is entirely indifferent to the location of our main three axes of reference in space, in other words, it is isotropic. Four

dimensional space-time is quasi-isotropic. It certainly pays no attention to the directions in which different observers count time. To any particular object, at some particular point in space-time, its experiences, which is the stuff it is made of, are strung out in a very distinct time order. But viewed from another object, which moves with reference to the first, the time axis as well as the three space axes are tilted so that time is transformed to space-dimensions and vice versa. There is therefore no absolute differentiation between time direction and space direction any more than between horizontal and vertical in our ship analogy. Space-time is in truth a four-dimensional continuum not a three-dimensional space with time in some manner superimposed. *When time direction as well as space directions are viewed under oblique angles their divisions, say in unit lengths, naturally change size like the height of the tilted ship.* We can already now explain one of the beforementioned puzzles of science. Two famous experiments, performed with great accuracy, seemed to indicate, one that the aether is not carried along with moving bodies but passes or "blows" freely through the interstices of matter. The other proved just the reverse. To account for this baffling result many ingenious theories were offered, the most plausible of which was that all matter in motion suffers a contraction in the direction of its motion and the amount of this contraction was arbitrarily chosen so as to account for the discrepancy. The relativist who believes in four-dimensional space-time calculates the contraction due to the tilting of the axes involved in the experiments and finds that the contraction necessarily must have the value which reconciles the experiments.

It is interesting to note just how large the contraction of the earth must be if viewed from the sun. The theory gives a contraction in the ratio $\sqrt{1-(V^2/C^2)}$ where "C" is the velocity of light and "V" the velocity of the object rela-

tive to the observer. Inserting values for the velocities of light and of the earth relative to the sun, we find a contraction of two and one-half inches on the diameter of the earth. This is a fair example of the magnitude of the corrections that the theory of relativity imposes on the prevalent laws of mechanics and yet what a revolution of thought it introduces.

If we examine the expression $\sqrt{1 - (V^2/C^2)}$ we notice that if "V" equals "C" the whole term becomes zero, in other words an object passing the observer with the velocity of light would appear to have no length, it would have shrunk to a surface. In reality this could not happen because it would take an infinite force to impart light velocity to a particle of matter. This velocity is therefore the highest possible velocity in the universe, that man at present can conceive, and it is furthermore, or rather therefore, "absolute," *i.e.* all observers agree on its value, if measured in vacuum and sufficiently far from gravitational influences.

But are these changes in time and space "real"? A futile question. We cannot say that any point of observation in the universe has preference over any other because we do not know the absolute motion of any point. If we had found an inflexible, unyielding aether-wind blowing through space and matter we might have assumed the aether as standing still and chosen units of time and space measured along reference axes drawn in the aether as our absolute standards. We might then perhaps have permanently separated space and time. As the aether is superflous in the world of relativity, the relativist is indifferent as to whether the aether hypothesis is retained or not.

But we face again the question: how can we describe nature if all our measurements are relative, dimensions and duration depending on the motion of the observer? So far we have only found the velocity of light to be "absolute" under certain conditions. If, however, we follow an object

along its *worldline, i.e.* along its track through space-time, we should be able to make a record of the "distance" we travel, each unit measure along the wordline advancing in both time and space, and make notations of the place and moment at which our track crosses other worldlines, that is note the time and place of all the events that constitute our experience. The time thus noted is called the "proper time" for the worldline followed. The "distance" between two events is called an *interval* and here we meet again an absolutee quantity independent of the point of observation. We share in the motion of all objects on earth to such an extent that the necessity for measuring intervals instead of length and time separately does not occur in our practical life until we commence to explore the stellar spaces or observe particles moving with velocities approaching that of light as for instance particles shot out by radio-active substances. In these macrocosmic and microcosmic worlds, however, we must measure intervals because we are unable to follow the observed objects in their own tracks and are obliged to use "local" space and time directions and we know now that such measurements have no direct application on the object moving relatively to us. The mathematician, however, is able to calculate the interval between two world-points on a worldline from "local" observations much as a surveyor calculates the distance between two mountain tops, that he cannot reach.

Intervals are absolute, *i.e.* the worldline in space-time form a net-work whose location in space-time is independent of the observer. The intervals from knot to knot, *i.e.* from event to event, are not straight. They would be straight for an object moving with uniform velocity in a straight line in three-dimensional space or which is the same, moving with uniform velocity in a space free from other matter. Such motions do not exist in nature. All motions within our experience are curved in three-dimensional space and

very frequently the velocity in the curved path is not uniform but accelerated or retarded. In any case their track in space-time is curved. The worldline of the earth is essentially a spiral, a loop in space pulled out in time to such an extent that the windings are hardly discernible when viewed at right angle to the path. We see a closed orbit only when looking along the time axis so that the windings are superimposed upon each other. In similar manner most worldlines approach straight lines. Why should they curve at all? The curvature in the orbits of the planets in space is, according to Newton, due to a mysterious force, gravitation. Of the nature of gravitation we know literally nothing except that it follows the mathematical law, formulated by Newton, and long considered the model of all natural laws. Newton's well-known law of gravitation states that two material bodies attract each other with a force directly proportional to their masses and inversely proportional to the square of their distance. The mass of a body, or the quantity of weighable matter it contains, has been considered an absolutely unchangeable constant. It has lately been found that this is not strictly true. Particles shot out by radio-active matter have been proved to possess a mass increasing with the square of their velocity. Their change of mass was measurable because their velocity approached that of light, 186,000 miles per second, while the speed of a rifle bullet is less than one mile per second and therefore the change of mass with velocity has entirely escaped our notice until the radio-active substances were discovered. This change of mass with velocity leads to the amazing conclusion, also supported by electro-magnetic theory, that *all mass is due to motion,* whether it be the motion apparent to our senses or the motion with which our intellect has filled the "interior" of matter. But "what" moves there? The "what" must

itself be motion, if it has mass, so that matter, if thought to mean substance, utterly vanishes before our mind.

We see therefore that when we speak of mass in Newton's law we do not accurately know what mass we are talking about. But neither are we able to say what the distance between the two moving stellar bodies is. For what point of observation shall we choose? Thus Newton's law has become ambiguous although the variations involved are of insignificant magnitude as long as we deal with ordinary velocities. It also leads to one of those puzzles of science that relativity delights to explain. The orbit of the planet Mercury is an ellipse the major axis or largest diameter of which should always remain parallel to itself in space according to Newtonian mechanics and except for deviations already known to science. Astronomers have, however, observed a slow turning of this axis amounting to about 42 seconds of arc per century which has no known cause. The accuracy of this observation is so great that the probable error could not be as much as two seconds. Although we have not yet seen in what manner the theory of relativity replaces Newton's law, we might state already now that it gives a value of 43 seconds per century to the turning motion mentioned, one of the most remarkable verifications of a theory in the history of science.

We cannot state our new law of gravitation until we have given a little further study to the quantity mass. We meet it not only in the problem of gravitation but whenever we start or stop, change the velocity or direction of a moving body. When the elevator starts upward we feel an increased pressure from the floor; when the train starts or stops we lurch backward or forward as if there was a magnet pulling us in a direction opposing our change of motion. When the elevator starts upward it is as if our mass or that of the earth had increased. But we know the phenomenon is the result of our acceleration only. And again we

might remove the earth altogether, *i.e.* abolish gravity in the elevator provided we increase our "upward" acceleration with 32 feet per second which is the change in velocity due to gravity. We would then be entirely ignorant of the removal of the earth as far as anything inside the elevator was concerned. In fact, all the phenomena due to an acceleration may be identically reproduced by introduction of a suitable mass *and in small regions the reverse is also true,* as in the case of the elevator. This is the *principle of equivalence* which plays such an important part in the theory of relativity. We see now that by studying acceleration we study gravitation as well. It is a remarkable fact that the entire theory of relativity is with logical necessity developed from the two postulates propounded by Einstein: 1. The constant velocity of light in vacuum. (Along its worldline light does not travel with constant velocity in the neighborhood of matter.) 2. The principle of equivalence.

Let us now look at gravitation and acceleration from the relativist's point of view. We shall first see that mass is relative just as length and time are, *i.e.* dependent on the motion of the observer. This follows directly from the fact that mass varies with velocity and time enters into any statement of velocity. The observer uses his "proper time," *i.e.* measures time along the direction of his "local" time-axis in space-time which is tilted with reference to the local time-axis at the worldline of the moving body and therefore he obtains local time and consequently also local mass. The theory of relativity gives exactly the change of mass with speed found in radioactivity. Again we need not despair less the absolute has disappeared, for mass multiplied with time, which we call action, is absolute, *i.e.* independent of the observer, and gives us therefore an exact basis for our calculations.

In Einstein's law of gravitation "local" 'mass and "local" space-measures replace the "absolute" quantities in New-

ton's law. But, more important yet, the properties of space-time, as revealed by the study of accelerations, explain the curvature of worldlines, without resorting to mysterious attracting forces between distant bodies.

Worldlines curve for the same reason which causes a ball rolling on an uneven surface to change direction according to the inclinations of the surface. Four-dimensional space-time is grooved along the tracks of matter. This is what is meant by the statement that space-time is "non-euclidian." Masses must follow these grooves and the theory also shows that in doing so they fulfill the well-known principle in physics of *least action.*

Popular statements of the "relativity" law of gravitation generally introduce at this point the term "geodesic," *i.e.* the shortest route or interval length between two world-points. These geodesics are not straight lines in non-euclidian space-time and by stating that moving bodies follow geodesics the curvature of their path is supposedly explained. This is not strictly true. Let us take an example from our three-dimensional world, setting time aside for the present. If we are to move from New York to San Francisco the shortest route is through the earth, 2,555 miles long, and we would pass Omaha 204 miles below the earth's surface. If the earth were homogeneous, *i.e.* analogous to "euclidian" space, we would of course follow the straight line everywhere. But as we are apt to strike strata of different hardness, *i.e.* finding our space "non-euclidian," we might make better progress by dodging the hardest spots although thus lengthening our course. In that manner we might find many routes equally entitled to the claim of being the "shortest," *i.e.* to the name of "geodesic" and so we might hesitate which road to choose. But nature never hesitates; there is always some *unique* way of doing a thing and that way is always the choice of nature. With our "superior" intelligence we would prob-

ably suggest that the shortest line from New York to San Francisco is the geodesic of the surface of the earth which joins the two points and is 2,602 miles long; but the German cannon balls would never have reached Paris if they had shot through the air according to the rules of ordinary ballistics; it was only by searching the upper, rarefied layers of air, over a longer route, that the goal could be reached. To those cannon balls "air-space" was non-euclidian and the curvature of their path was governed by the law of "least action." And so in our four-dimensional world moving bodies find their "unique" path not along the shortest possible interval lengths but, peculiarly enough, "least action" is obtained along the *longest possible* interval lengths. (Our analogy is pictorial only and would be entirely misleading if rigorously applied.) Matter is building up space-time as various rocks and strata are building up the volume of the earth and as the various strata of air are building up the atmosphere, so that bodies moving in space-time follow curved paths as the worm in the earth, the road in the hills and the super-cannon ball in the air.

The network of worldlines is therefore distorted in the vicinity of matter. This property of space-time is difficult to conceive but no more so than the mysterious property of gravitational action on distance which the Newtonian theory asks us to believe in. The greater probability of the theory of relativity rests on the fact that it follows with mathematical necessity from the properties of space-time and the principle of equivalence and that its formulas give our present, less precise, laws, in a first approximation, and take care of their shortcomings in a second approximation. Like all theories it must be verified by experience and we have already seen that it accounts for many puzzles and anomalies of orthodox science. We shall now consider the most triumphal verification thereof. We have stated that space-time is grooved along the tracks of matter so that

other worldlines entering such regions must follow the grooves. This must apply to rays of light as well as to ponderable matter. Einstein therefore predicted that a ray of light coming from a distant star would be bent if grazing the disk of the sun on its way to an earthly observer. There is nothing substantial about light but because it exerts pressure on the surface it hits and because radioactivity early had shown that mass may exist by virtue of velocity alone, it was admitted that a ray of light might possibly bend in accordance with Newton's theory of gravitation when passing the sun. The theory of relativity, however, shows that there are two kinds of distortion in space-time around matter, one major, which in the main accounts for the Newtonian law of gravitation, and one minor, confined to space alone, which as a rule is insignificant even for stellar velocities. When we deal with the velocity of light, however, this second distortion rises to the same magnitude as the first and Einstein therefore predicted a deviation from a straight line just twice as large as could be expected if light, indeed, followed the Newtonian law. It is now a matter of common knowledge that the experiment was made during the last solar eclipse on May 29, 1919, and that even conservative scientists had to decide in favor of Einstein.

If therefore we are justified in believing in four-dimensional space-time, *almost* "flat" in *almost* empty regions, and curved where matter is present, we are able to draw a most inspiring picture of our universe. Who has not wondered what is "beyond the stars." If we consider a sufficiently large region of the universe we may say with fair accuracy that matter is on the whole uniformly scattered throughout the region. Each particle of matter contributes a small curvature to space-time so that a ray of light following the path of least resistance, *i.e.* the natural curvature, will be, not smoothly, but continuously bent. Ulti-

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mately it must return to its source. The lightpulse will then have explored the entire space-time continuum and found that it is closed. It does not matter in which direction it starts out it always ultimately comes back again. It is therefore possible to explore every point of the universe and we must draw the conclusion that the universe is finite but on the other hand has no boundary. "Beyond the stars" simply does not exist. The difficulty we find in picturing such a universe to ourselves need not deter us from embracing the dogma for—is it easier to understand an infinite world? The best analogy found for closed space-time is the surface of a sphere. Flat beings confined to such a surface and unacquainted with the vertical so that they perceive length and width only must live in the faith of our own ancestors that their world is flat and must, like they, ponder whether it is infinite, whatever that may mean, or has boundaries, and in such case what there is in the great beyond. Imagine their bewilderment if it were proven that their world was finite and yet without boundaries. In whatever direction you started out you would eventually come back. In such a world every point is of equal importance and is a definite part of the whole in fact it is symmetrically placed with reference to the whole. So in our space-time every "event" marks in a sense a "central" point which cannot be disregarded by comparison to the infinite but is indeed an integral part of a definite scheme and interlaced through its worldline with every other part of the entire universe.

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MEMORY AS KNOWLEDGE OF THE PAST.

IN this paper, I propose to discuss memory as it interests the student of the theory of knowledge rather than the psychologist. This does not mean that I intend to disregard psychological considerations but I shall deal with them only in so far as they affect the cognitive problem—and it cannot be denied that they do affect it. This paper divides itself broadly into four parts: (a) preliminary definition of memory; (b) data concerning memory; (c) analytic definition of memory; and (d) corollaries and conclusion.

The word "memory" has at least two meanings: in the first place, it may mean the mere ability to repeat, in the second, it may mean a way of knowing. An example of the first is expressed in such a phrase as, "I remember the poem," meaning "I can recite it," or in "I remember the song," meaning "I can sing it." Memory, in this sense, denotes the presence of a habit, of a disposition which, when stimulated, starts a certain semi-reflex motor process. An illustration of the use of our term in its second sense is found in such phrases as, "I can recall the details of the boat-race," or "I still remember the look on his face," where the word memory is used to denote a certain form of apprehension. As in this paper our problem is cognitive, we shall confine ourselves to memory taken in the second of the above senses.

Parallel to this ambiguity in the word memory is an ambiguity in the word knowledge. We use the term knowl-

edge to mean an event strictly relevant to epistemology as when one says, "I know that two and two make four;" but we also use it to mean situations quite irrelevant to the theory of knowledge, as for example, when one says, "I know how to play tennis," meaning, "I can play tennis," or "I know how to deal with the fellow," where the term knowledge refers to ability for a certain sort of behavior. I should add that the use of the word knowledge in the sense of *kennen* (as contrasted with *wissen*) is use of it in the latter sense; thus, "I know London," means "I can find way through the streets," or 'I know French," means "I can speak or understand French." But this is a digression.

To return to our topic, this second sense of memory—as a form of knowledge—is still too ambiguous. In this paper, I will use the term in the particular sense in which to remember is *to recover a perceptual impression.* This restricts memory to knowledge of the physical world through an image. Conversely, it excludes (a) all abstract memory such as, on the one hand, memory of formal, mathematical truths, and, on the other, of mythical objects like dragons and other intellectual constructions (as when the artist recalls a vision he formed in his mind the previous day). And (b) it further rules out all memory of the physical world which consists merely in a belief and involves no image of the event remembered; in particular it excludes, on the one hand, memory based on information, e. g. when I remember that Japan is an island (having learned it from my geography)—and, on the other, even memory based on perception when it is not actually a recovery of the impression, e. g. when I remember that John is tall because having seen him I have the knowledge that he is tall. It is obvious that the qualification we have specified above excludes these types of memory, since what

is recovered in these cases is either a previous idea or a previous belief but not a previous perceptual impression.

I have thus limited my field of inquiry to memory in the sense of such cognitive facts as are a recovery of a perceptual impression, in other words, to knowledge of the physical world by means of an image. This type of fact is best illustrated by the state of mind of the eye-witness when, in his testimony, he tries to recall the details of a street-accident. Ours is the special sense in which memory has been debated in epistemology for only in this sense does memory offer any genuinely distinctive features.

What are the "hard" data concerning memory?

(1) Memory is the immediate apprehension of an image.

(2) Further, it is in some way knowledge of the real world, a reference to an object "out there." In memory, while apprehending the image I am at the same time apprehending my friend John or Mont Blanc. Whereas in imagination the image is just an image, in memory the image acquires the tang of reality, in that it seems to be part of the physical world, or at least, to represent it.

It follows that psychological analysis is unable to exhaust the nature of memory. For, to define memory entails a double task: first, to analyze the mental event—a task appropriate to psychology; and secondly, to ascertain the relation of the image to the external world, or to define the sense in which memory is good or bad knowledge—a task falling outside psychology and within epistemology.

(3) Memory is knowledge of the *past*, but only of that portion of it which has been already experienced by the subject.

(4) Memory is *primitive* knowledge of the past. Unlike historical knowledge and like perception it is not logically derived from other knowledge.[1] Memory is therefore

[1] Memory of course presupposes perception but the relation is strictly causal; memory is not *inferred* from perception.

one of the sources of knowledge and, indeed, the chief, if not the only, source of our knowledge of the past. In view of the primitive character of memory-knowledge, no good analogy can be established between it and "knowledge" or anticipation of the future; knowledge of the future, such as it may be, is inferred.

(5) Memory is fallible and it would perhaps be more accurate to call memory opinion rather than knowledge. Moreover, memory is a matter of degree; it is clear or vague, faint or vivid, partial or complete.

The above are the most important data concerning memory—the facts that any theory of memory has to explain and to which it has to conform.

We shall now proceed with the psychological analysis of memory. What are the elements which compose it? (1) As has been already pointed out, memory is the apprehension of an image. (2) It is also a form of thought. In sense-perception, psychologists trace what is perhaps a fictitious transition from mere sensation to perception—from a condition in which this or that datum is sensed to one in which the sense-datum, through association or otherwise, takes on meaning; we then have what is called perception of an object. So too, memory represents a stage beyond mere imaging—one in which the image has already become the nucleus of a meaning; in memory we have the thought of an object. Generally speaking, the relation between the datum and the object which it means is of two kinds. Either the datum is similar to its object as in the case of photographs and hieroglyphic language, or it is not, as is true of the word table (written or imaged) standing for the object table, and of all conventional language. Memory is an instance of the first variety. If, in memory, we are thinking of a certain table, then our datum—the image—is a picture of the table. Let us say then that memory is an instance of *pictorial* meaning. (3) But in imagin-

ation, too, we have images with pictorial meaning—images of people and of landscapes. To distinguish memory from imagination, we must introduce another element, namely, the sense of reality, which, in memory, attaches to the image. To mention an analogy from the "movies," imagination is like our state of mind when following a story of the Wild West on the screen while memory is like our attitude when watching the Pathé pictures of current events. In the words of James[2] memory involves "a feeling of belief," which may become explicit in the judgment: 'this is an image of the past."

(4) The following example will show that our analysis of memory is still incomplete. I failed, let us suppose, to attend a football game in which I was much interested. I read a good description of it in the newspaper and, aided by my excellent visual imagery, I form in my mind a clear image which I believe to represent accurately what transpired at the game. My state of mind includes both ideation and belief and yet clearly is not memory. Memory involves something in addition, to wit, the feeling of familiarity or what Titchener calls the recognitive consciousness; this attaches to the memory-image and, on occasions is rendered explicit in the judgment: "I have perceived this before."

These, then, are the four atoms of which memory—the molecule—consists: apprehension of an image, meaning, feeling of belief, and feeling of familiarity. Taking the first and the second pairs respectively, each as a unit, we may define memory as a mental state consisting, on the one hand, of ideation of the pictorial sort, and, on the other, of the molecular judgment: "this is an image of a past event that I have perceived."

[2] *Psychology*, Vol. I, p. 652.

But consider the following example. A is shown a picture of B—an old acquaintance now totally forgotten—and thereupon forms in his mind an image of B, at the same time saying to himself: "So this is B that I knew so well." A's state of mind includes both ideation and the appropriate judgment referring B to A's past; nevertheless, it is certainly not memory. This is because, though A's judgment is similar with a true memory-judgment in content, it lacks the character of logical primitiveness belonging to the latter. A came to know that his image is a picture of B, only because he was told that the photo to which his image corresponds is a photo of B. We should thereupon modify our definition of memory to read as follows: Memory consists on the one hand, of the element of pictorial ideation, and, on the other, of a *primitive* judgment to the effect that the ideate is a past event already perceived by the subject. Note that the judgment need not be explicit; it may be only a feeling, it may even be just a motor set, a readjustment of the nervous mechanism preparatory to action. It is precisely this fact that in the last analysis distinguishes memory from other mental events involving an image, like fancy and the association of ideas—namely, that in memory the apprehension of the image goes with a process of central readjustment which is absent from other analogous mental events. In this respect, memory compares with perception, except that in perception the readjustment is peripheral as well as central.

Finally, of the two elements in memory, the judgment is invariable whereas the element of ideation varies in content with different memory-acts.

So much for the psychological analysis of memory. I will now consider memory in its purely cognitive aspect, as knowledge of the past.

Two views present themselves: one that memory is knowledge of the past by *acquaintance,* the other, that it is knowledge of the past by *description.* The first has several points in its favor. First of all, it fits into appearances. Memory is *some* sort of immediate apprehension—and since the past is its object—why not an immediate apprehension of the past? And then, this view explains matters in a simple fashion. If memory is primitive knowledge of the past, why—it would argue—that is because memory is an immediate contact with the past. And lastly, the theory of memory as acquaintance with the past establishes a definite analogy between memory and perception. The real world is conceived as immediately given in both—in perception, the portion of it which is present, in memory, that which is past. In sum, this theory would maintain that in recalling, *e.g.* one's childhood days at the family hearth, it is the very situation itself, the old room and the grandmother now dead, that appear before one's mind.

The above view is confronted with serious difficulties which far outweigh its merits. (1) In memory one often checks up one's image, judging it to be a good or a bad picture of the object. Therefore, image and object are not the same; in other words, what one apprehends immediately in memory is not the past. (2) One often passes from the content of imagination to that of memory without any hitch. For example, I may suddenly recognize the lovely scenery that I am conjuring in my fancy to be Yosemite Park which I visited last year. It is thus possible for the datum of imagination and the datum of memory to be one and the same thing, and if, as seems clear, the former is not a member of the physical world, neither is the latter. (3) Experience has shown memory to be fallible—not merely in respect of the inferences drawn from it, but in itself; but acquaintance does not admit of error.

It appears then that memory is not knowledge of the

past by acquaintance. Presumably, it is knowledge of it by description. An object is said to be known by description when (a) it is not directly apprehended and (b) is referred to ambiguously or definitely, in terms of some one of its properties or relations, as "a so-and-so" or 'the so-and-so." E. g. I am not acquainted with Milton; but he happens to have written *Paradise Lost* and I refer to him in those terms, as the author of *Paradise Lost*. Turn to memory. That its object—the past—is not directly apprehended has been just shown; that further, it is referred to in terms of some description is indicated by a circumstance noted earlier. We have seen that over and above the image, memory involves a judgment to the following effect: "this is an image of a past event," or "there is a past event similar to this image." To all intents and purposes, this judgment is a reference to the past in terms of the property which the latter possesses of being similar to the image. We conclude that memory is knowledge of the past by description, and, more specifically, knowledge of the past *as the object pictured by the image in the mind.*

To argue that memory is knowledge by description and not by acquaintance is also to maintain that memory is representative and not presentative knowledge. Memory is what some idealists have claimed all knowledge to be. that is to say, knowledge of the external world through an idea which is a copy of the real object. Moreover, memory constitutes a perfect application of the correspondence-theory of truth, for memory is true or false according as the image reproduces the object well or badly.

But to return to our definition of memory as descriptive knowledge. Is memory definite or ambiguous knowledge of the past? That depends upon the circumstances. In recalling the face of my room-mate, I refer to a definite object, my judgment being of the form: "there is one and only one object pictured by 'this' image, and it is the face

of my room-mate." But consider my state of mind in recalling childhood scenes, e. g., the family worship every morning. In this instance, my image is really a composite of my various impressions of the family prayers, and through the image, I am referring ambiguously to *any one* of the instances of family worship in my past.

Let us return to the data set down in the beginning of this paper, attempting to interpret them in the light of the preceding analyses and also glancing at further facts concerning memory.

(1) The memory-act has a definite purpose: to provide knowledge in the form of some assertion of fact. For example, by recalling to mind the scene of yesterday's dinner and after scrutinizing the image, I am led to exclaim: "Yes, it was plum-pudding we had for desert last night." How is this knowledge extracted from the memory-act? To get an answer we need only put to ourselves another question concerning a similar case: How is it that when I hear the telephone ring, I know that somebody wants to talk to me? Obviously, because the telephone-ring is a sign with a definite meaning. So too, the memory-image is a pictorial symbol of a past event, and it is in so far as I give to the symbol some definite interpretation that I am enabled to proceed to an assertion of fact. This process has two stages. In the first, I recognize the object; thus, on apprehending the image, I say to myself, "yesterday's dinner-table!" In the second, I go on to scrutinize the image, meanwhile making appropriate assertions concerning the object recognized, e. g., "the table was decorated with flowers," "it was a round table," etc. And as the image becomes clearer in my mind, I keep making new assertions in ever-increasing number.

(2) *Vagueness.* At least three varieties of vagueness may be distinguished in memory. (a) The first—to which we made an allusion in the paragraph on memory as de-

scriptive knowledge—consists in an ambiguity of reference. An example is the case of the old graduate recalling the class reunions where he often enjoyed himself in the past. In all probability, he has one image in his mind, standing indifferently for any one of these reunions. (b) The second variety consists in an indefiniteness in the recognitive element of memory, e. g., it often happens that we have in our minds an image of a person whom we are sure we have seen before, and yet we cannot recognize him. Habit-memory offers the familiar analogy of tunes persistently haunting our minds without disclosing their identity.

Varieties (a) and (b) may be contrasted as objective vs. subjective ambiguity respectively; in the first case, the vagueness lies in the reference, whereas in the second, the reference is perfectly definite—we are simply unable to locate its term.

(c) The third variety consists in vagueness of the image—vagueness which might prevent me from judging, e. g. concerning the dinner table, whether it was round or square, just as, similarly, vagueness in a photograph might prevent me from judging whether the person pictured had a moustache or not.

Now, we saw that in memory the image serves as a pictorial symbol of the past. We can therefore proceed to characterize vagueness in the first two cases as indefiniteness (objective or subjective) in the *denotation* of the symbol, and vagueness of the third variety as indefiniteness in the *connotation* of the symbol. In cases (a) and (b) we fail to associate the image with any particular object, whereas in case (c) we fail to associate it with any particular meaning.

(3) *Error in memory.* The occasions of error in memory are, I think, twofold. First, the image may be an inaccurate picture of the past or even may not represent anything real at all; secondly, the image may be accurate but

vague. If the first, the memory-judgment: "this is an image of a past event" would naturally be false, as also the assertion of fact based upon the interpretation of the image. If the second, the vagueness of the image might lead to error in the interpretation and thus ultimately to error in the assertion.

(4) We have seen that to remember is to recover a previous impression; what is the relation of the memory--image to the perceptual impression? The image may be (a) simple, corresponding to a single impression, or (b) may be composite, like a composite photo, corresponding to a group of previous impressions. In the first case, it is a plain picture of the object, but in the second it is more like a portrait than a photo of it.

(5) In past discussions of memory, it has been suggested that whenever we remember an event, we are really remembering our having perceived the event. This is not borne out by the facts. I have at this moment an image of Hyde Park, London, which I visited some years ago, without at the same time, having an image of myself seeing Hyde Park. And when the image is composite, clearly, it is well nigh impossible to associate it with any particular impression in the past. Nevertheless, a feeling of familiarity does accompany the image of the event remembered, enabling the subject to "place" the event in his biography—especially if the feeling is fairly definite. In other words, though the event remembered is not necessarily accompanied by the memory of one's having experienced it, it remains true that the event is, so to speak, appropriated by the subject and recognized as part of one's experience.

It is almost impossible to summarize a paper of this nature but several general impressions emerging from the discussion may be recorded. Memory may be described in various ways but, above everything, it is knowledge of the past. Moreover, it is primitive knowledge and there-

fore along with perception and *a priori* knowledge, deserves to be classed among the sources of human knowledge. But memory is fallible and perhaps should on that account be called, in Platonic language, a form of opinion rather than knowledge.

Memory constitutes a double problem, psychological and cognitive; it may be studied as a mental process, or as knowledge of the real world. Approaching it from the first angle, we found memory to be a judgment concerning an image, roughly speaking. More strictly, we found it to consist, on the one hand, of a process of ideation with an image as its nucleus and on the other, of a judgment to the effect that the image is a picture of a past event already perceived by the subject. Approaching memory as a cognitive fact, we found it to be knowledge of the past not by acquaintance but by description, namely, as that which is pictured by the memory-image. We were thus justified in characterizing memory as a type of representative knowledge, the memory-image as a pictorial symbol of fact, and the process of making assertions on the basis of the memory-act as one of interpreting the meaning of the symbol.

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THE AESTHETIC CATEGORIES.

THE term beauty is sometimes taken very broadly so as to cover the entire field of aesthetic excellence, as by Croce and Carritt.[1] But this use stretches it far beyond its normal meaning; it is better to recognize other coordinate terms for aesthetic excellence. The most notable of these supplementary terms is the sublime. To these the tragic and the comic are often added. Even these will scarcely suffice if we are to find an appropriate category for the genre paintings of Teniers or the novels of Dickens.

If our analysis were to move in the plane of literary usage, the categories would multiply beyond control. Beauty would need to be distinguished from charm, loveliness, delicacy, elegance, splendor, and many such terms; sublimity would invite a comparison with grandeur, impressiveness, and the like. We should encounter a tangle of synonyms leading to endless discriminations in the manner of Prodicus. Let us then abandon the dictionary method, and seek some more definite instrument of classification.

The aesthetic field varies in many ways and is subject to multiple classification. But one of the most important ways in which it varies is in its appeal to specific human instincts. Let us take as our guide the analysis of human instincts worked out by McDougall and refined upon by Thorndike. In this paper I shall assume that the reader is familiar with the views expressed in McDougall's Social

[1] B. Croce, *Aesthetics*, ch. XII; E. F. Carritt, *Theory of Beauty*, ch. IX.

Psychology, which I shall follow, without, however, impugning the validity of Thorndike's more minute analysis; for psychology it may be necessary to split up such an instinct as fear into more specific fears of this and that, but for aesthetics the broader grouping would in any case remain more significant. Our task then is to consider the phases of aesthetic experience as they appeal to the various instincts and propensities of our nature, and to correlate this analysis with the traditional categories.[2]

First of all it should be noticed that there is a formal factor in aesthetic experience which makes little or no appeal to any instinct. This factor is dominant in non-pictorial designs, including Oriental rugs (when their symbolism is unknown or unheeded), jewelry in geometrical patterns, and stained glass (in cases where the representative element is negligible); it is also dominant in those musical compositions in which the emotional effect is minimal or absent, and an abstract pattern of agreeable tones is presented. To be sure a fairly plausible case can be made out for the doctrine that there is no purely abstract design for eye or ear—a line would always be an emotional gesture or a suggestion of the beloved lady's neck, while beauty of sound would be an echo of her voice or else of the shouts of victory. However, the relations of instincts to abstract design and abstract music are subtle and dubious, while our preferences in this field are immediate and vigorous. It seems safer to admit that the function of instinct in these cases is a vanishing quantity. Their beauty lies in their harmony with our senses and with our perceptive processes. Hildebrand and Cornelius have elaborated a theory of the ways in which perception is facilitated by good design; the same can be shown of the musical composer's devices of repetition, variation, contrast, and climax. Our first cate-

[2] *Cf.* the analogous ethical problem worked out by Prof. W. K. Wright in "The Evolution of Values from Instincts," *Philosophical Review*, XXIV, 1915, pp. 166-183.

gory then shall be sensory-perceptual beauty—or, if you like, formal beauty, involving little or no appeal to instinct.

There is no denying that the beauty of human beings is intimately related to sex, but the precise nature and scope of the relation is debatable. Both sexes are most beautiful at the age when they would most naturally woo or be wooed, and in those conditions of health and freedom from strain which are most propitious to wooing. There is a reciprocal relation between beauty and sex attraction. A woman's charm is enchanced by the abstract beauty of her form, coloring, and voice; it would be absurd to maintain that if all women looked like Hille Bobbe in Franz Hals's picture and spoke with strident voices, their attractive power would be no whit lessened. Conversely the beauty of painting, sculpture, and poetry is enhanced by incorporating sexually attractive features; it would be nonsense to assert that Botticelli's Venus or Keats' last sonnet would be as beautiful to sexless beings as they are to us. Our second category then shall be formal beauty enriched by sexual charm.

Another instinct which enriches aesthetic experience is the parental instinct with its concomitant tender emotion. No object is more fascinating to a mother that her child. She is enraptured by its tender and delicately tinted skin, its dainty little hands and feet. The impulse, being a protective and solicitous one, finds a charm in the very helplessness and fragility of its object. This delight in tenderness, delicacy, and fragility spreads into the aesthetic sphere. As McDougall says, 'It is amusing to observe how, in those women in whom the instinct is strong, it is apt to be excited, owing to the subtle working of similarity, by any and every object that is small and delicate of its kind—a very small cup or chair, or book, or what not." (*Social Psychology*, p. 74.) The fact that the taste for smallness and delicacy in artistic objects is more marked in women

than in men confirms its relation to the parental instinct, which is primarily maternal rather than paternal. The beauty of flowers and birds, of fragile Chinese porcelain and Venetian glass, and of fine-spun tracery in Gothic windows, owes much to this protective impulse. Our third category then shall be formal beauty enriched by the parental instinct with its tender emotion.

There is no incompatibility between the two preceding. Rather they tend to go together. In practical life, as McDougall has emphasized, sex attraction normally evokes also the tender and protective impulse. Similarly in plastic art and poetry the ideal of feminine beauty tends to include a delicacy or even a fragility that appeals to the protective impulse. Botticelli's Venus rising from the sea unites the appeals of sex and childlike tenderness, but the baleful flower-clad figure in his "Spring" appeals to sex alone. In Wagner's Tannhaeuser the charms of Elizabeth and Venus differ in the same way. Formal beauty may therefore be enriched by sex and tenderness together, or by either alone.

One further impulse needs to be aroused by all forms of beauty properly so-called—namely that of self-abasement. Without a tinge of humility the sense of beauty is incomplete. When it is absent we call the object merely pretty, not beautiful. A doll-like face may have sensory-perceptual agreeableness of color and form, but in default of individuality, depth of character, novelty or subtlety, it will seem empty or trivial. The faces that appear on the covers of cheap magazines are pretty—perfect in contour and complexion, with liquid eyes rolled in a sentimental appeal to sex and tenderness, but devoid of the depth and subtlety which alone can command the deference of the discriminating. These faces appeal to the *un*discriminating public—which also makes no distinction between "beauty" and "prettiness" in its praises.

There are various means by which the object can command our deference. The suggestion of a rich inner life may do it, as the poise of the Venus of Melos, the reverie of the Hermes of Praxiteles, the subtlety of Mona Lisa, the vital exuberance of Rubens's bacchanalian figures. Preciousness may do it, as in silk, gems, marble, and porphyry, in contrast with cotton, paste, and plaster. So may delicacy of workmanship, as in intricate carving and lace.[3]

This need of humility involves a slight reinterpretation of our category of formal beauty. It is formal in that there need be no depiction or suggestion of the object of any instinct, yet the fact of rarity, the suggestion of skill, or the impression of subtlety may serve to evoke humility.

The impulses of self-assertion and of fear both enter into the sense of sublimity. The former, however, is the primary factor, while the latter gives it a special tone by putting it to the supreme test. When the forces of nature terrify us so that we can think only of our own fear, we have no sense of sublimity, and the situation is a practical evil not an aesthetic good. It is only by imaginatively identifying ourselves with these forces and adopting their power as our own that we feel exalted and call the object sublime. The steep and gloomy mountain side, resisting the climber and threatening him with disaster if he fall, is sublime to us if, instead of being cowed by it, we read into it pride and strength and inhumanity, and exult in appropriating these to ourselves.[4]

Tragedy is the dramatic form of the sublime. Its central feature is the struggle of the hero with hostile forces; to these in most cases he succumbs, but only because it would have required more than human power to resist them. We

[3] Cf. what Santayana says of gems and of skilled workmanship in his *Sense of Beauty*, pp. 212, 213.

[4] Cf. Lipps, *Aesthetik*, Vol. I, p. 527 ff.; Santayana, *Sense of Beauty*, sect. 60).

are exalted by the hero's display of personal force, with which we sympathize and identify ourselves. The experience is rendered "a fearful joy" by the sense of impending or actual disaster, which, however, intensifies the element of self-assertion by straining it to the limit. Since tragedy involves struggle, our pugnacity is also brought into play, and wins a vicarious satisfaction.

The category of the comic is also connected with self-assertion and sometimes with fear. In spite of the violent contrast between the sublime and the ridiculous, the fact that it is but a step from the one to the other is so obvious as to be crystallized into a proverb. In both cases we find self-assertion contrasted with something formidable. In the sublime and the tragic, self-assertion is stimulated by a supreme test; in the comic it is liberated by the collapse of its opponent. Jokes are almost always "on" somebody or something, a weakness or defect of the victim being neatly shown up. It is still difficult to give a better account of our sense of the ridiculous than Hobbes's description of laughter—"sudden glory arising from some sudden conception of some eminency in ourselves, by comparison with the infirmity of others or with our own formerly."[5] Freud has given an illuminating supplement to this by showing how the process of self-discipline and social discipline leaves us full of latent hostilities to convention and to all persons and forces which lay claim to our deference. An easy victory to some aspect of the self is given by the humiliation of the pompous, the betrayal of stupidity, the subjection of solemn thoughts to verbal jugglery.

The relation of the sublime and the comic to formal

[5] Human Nature, ch. IX, sect. 14. Thorndike objects to this that it does not account for the happy laughter of playing children. But this too is usually evoked by some facile triumph—scuffing through autumn leaves, catching the opponent at tag, knocking down the tower of blocks, *cf.* H. M. Kallen, "The Aesthetic Principle in Comedy," *American Journal of Psychology,* XXII, 1911, pp. 137-157.

beauty demands consideration. Shall we say that the sublime is formless and the comic deformed? In many cases this seems to be true. Sublime mountain scenery is formless as compared with a geometrically ordered garden. The stars would be less sublime if reduced to a wallpaper pattern—or even to a religious motto.[6] In literature, however, the sublime usually has a high degree of formal beauty. This is true of the descriptions of God's power over nature in the Book of Job, the favorite passages in Paradise Lost, the song of the archangels in the prologue of Goethe's *Faust*. In painting, Michael Angelo's Creation of Adam has formal beauty as well as sublimity. It seems then that the sublime may dispense with formal beauty; indeed it may derive power from rendering perception difficult[7] (consider the starry sky) just as beauty derives charm from rendering perception easy; but in other cases it may form a splendid union with beauty. Burke recognized this union but did not dwell on it, and the names of "specious" or "fine" which he assigned to it were scarcely well chosen.

In the comic a similar situation emerges. Deformity is one of its sources; but in so far as this means merely departure from type it does not necessarily impair aesthetic unity: the Goops are as coherent as a correctly drawn human figure. Still I cannot think of any comic picture that could be called really beautiful. In literature the comic is compatible with a high degree of beauty, as in the witty speeches of Shakespeare's comedies, and in such verse as Lyly's lyric beginning "Cupid with my Campaspe played." The opera "Don Pasquale" envelops a farcical action in delicate and sprightly music; the product has exquisite beauty.

[6] Cf. Santayana, *Sense of Beauty*, p. 105.

[7] Cf. Bosanquet's distinction of facile and triumphant beauty in the third of his 'Three Lectures on Aesthetics."

The beautiful, the sublime, the tragic, the comic—these are the aesthetic categories most frequently discussed. Yet it is astonishing how much of the aesthetic realm falls outside them all. Genre paintings are ordinarily neither beautiful nor sublime, yet they have a value which is assuredly aesthetic rather than theoretical or practical. Romantic novels like Scott's are too diffuse for beauty and are tragic only in spots. Dickens is comic at times but that does not sum up his whole aesthetic value. We need a category that demands less concentrated coherence than beauty, less tension than sublimity; otherwise we shall be at a loss in dealing with a large field of aesthetic experience. The best term which occurs to me is "the interesting," which can well indicate an appeal to miscellaneous interests. Fiction and genre painting offer a vivid and sympathy-awaking depiction of miscellaneous aspects of life; this depiction appeals to similarly miscellaneous interests in us, and is therefore called interesting. To this category nothing is therefore called interesting. To this category nothing human is foreign.

"Greift nur hinein in's volle Menschenleben;
. . . . . .
Und wo ihr's packt, da ist's interessant."[8]

This does not mean, however, that one subject matter is just as good as another, for even Goethe's "lustige Person" would scarcely deny that interests vary in the profundity of their appeal and also in their adaptability to aesthetic treatment.

If there be a primitive tendency to the sympathetic induction of emotion it would appear to be involved in almost the whole field of the interesting. By sympathy here I mean the tendency to share the feelings and attitudes displayed by others. It would account for the fascination ex-

[8] *Faust, Vorspiel auf dem Theater.*

erted by narratives of human adventure and emotion, and would furnish the medium through which all our various instincts may be evoked. In practice sympathy tends to be limited to members of the same group. It has its counterpart in antipathy to intruders and to all beings which are suspected of hostility to the group; as regards them we delight in their distress and are perturbed at their delight, and their mere presence is a source of uneasiness. Antipathy too has an aesthetic function, for it may enhance our excitement and interest—the villain adds much to the play; but a work in which all the important characters are repellent is almost sure to be repellent itself.

Curiosity too plays a large role in the category of interest. Suspense of plot arouses it strongly; the desire "to see how it will come out" is much exploited in fiction and drama. The local color of voyages and historical novels appeals to curiosity, and the varieties of human character do so in an even higher degree.

Sympathy and curiosity then are fairly pervasive factors in the field of "the interesting" and help to give it unity.

On the borderline between interest and beauty lie works which lack the clear pattern of a lyric or a Greek tragedy, and make a strong appeal to varied interests, yet achieve beauty by a pervasive unity of fascinating atmosphere. Such are the Homeric epics, where the music of the verse, the nobility of sentiment, and the facility of the imagery create a charming unified atmosphere. Similarly in our own day Joseph Conrad adds beauty to interest; his style bathes a whole story in an atmosphere which is

"Simple, sensuous, passionate."

It is worth considering whether the opposites of the categories so far considered have any positive character. Beauty, at least, has a positive counterpart in ugliness.

One factor in ugliness is the antithesis of sensory-perceptual beauty: certain combinations of color or of sound seem to be fundamentally at odds with our perceptive apparatus; certain modes of design resist perception, as did the atrocities of the so-called *art nouveau*. Another source of ugliness is disgust, which McDougall regards as one of the primary instincts. Whatever is filthy or morbid tends to arouse this response; it will always be difficult to secure beauty in the depiction of dungheaps or disease. If antipathy is a primitive tendency it should be mentioned here. Whether primitive or not the sentiment of aversion to what is foreign is early and strongly developed. The foreigner is distrusted and disliked—though a specific taboo may make the guest sacred. Hence divergence from the facial and bodily type of one's own group is normally regarded as ugly. The cripple and the idiot are revolting in a different and more profound way because they are alien to humanity—yet not so alien as not to claim human rights. Under the influence of humanitarian ideals we may inhibit this antipathy but I imagine that very few can altogether get rid of it. The positive character of ugliness, which distinguishes it from mere absence of beauty, seems to be due in its various instances to one or more of the factors just mentioned: harshness to sense or perception, a tendency to evoke disgust or a tendency to evoke antipathy.

The sublime, the tragic, and the comic have no positive antitheses; we find merely the non-sublime, the non-tragic, the non-comic. Interest, however, seems to find a positive antithesis in dullness (personified by Pope as a goddess). A series or complex of stimuli, without excellence of form and without power to awake any particular instinct, produces languor, fatigue, or restlessness; thus dullness produces boredom and wins a positive status.

Our attempt to trace the influence of the various instincts on the aesthetic experience leads to the following

results. We have found a significant subdivision of the category of beauty, contrasting formal beauty with that which is colored by sex or tenderness, and noting the role of humility in marking off beauty from mere prettiness. In regard to the sublime and the comic we have emphasized the role of self-assertion, without minimizing the distinction between the tension of sublimity and the *détente* of the comic. In the tragic we have recognized a dramatic form of the sublime, in which our pugnacity also makes itself felt. Furthermore we have felt the need of a supplementary category, "the interesting" which appeals, chiefly through sympathy and curiosity, to an unlimited variety of interests. On the negative side we have found a positive basis for ugliness in harshness to sense or perception, and in the instincts of disgust and antipathy, while a positive basis for dullness appears in fatigue.

If the preceding train of thought led merely to a new classification it would scarcely be worth the trouble. But I think it has a positive value in showing how the aesthetic realm is responsive to our vital interests, so that our aesthetic experience becomes a precious supplement and reinforcement to the other activities of life.

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THOUGHT AND MENTAL IMAGE, ART AND IMITATION: A PARALLEL.

"CHOOSE a day on which to represent the most sublime and affecting tragedy we have, appoint the most favorite actors, spare no cost upon the scenes and decorations, unite the greatest efforts of poetry, painting and music; and when you have collected your audience, just at the moment when their minds are erect with expectation, let it be reported that a state criminal of high rank is on the point of being executed in the adjoining square; in a moment the emptiness of the theatre would demonstrate the comparative weakness of the imitative arts, and proclaim the triumph of real sympathy."[1] Burke's insistence, in this passage, on the superior claim made upon interest or emotion by the real event contrasts, or seems to contrast, with Durkheim's discovery that among Australian aboriginal tribes the images of totemic beings are more sacred than the beings themselves.[2] A reality can detach interest from a mere representation if Burke's estimate be right; a symbol can excite more emotion than the reality it represents if Durkheim's statement be true. The sides of this antithesis are not in even balance: a real execution does not contrast with a stage-play precisely as a sacred being contrasts with its own symbol nor are spectators in a London theatre Australian aborigines. But Burke himself says

[1] *An Essay on the Sublime and Beautiful*, 1-15.
[2] *The Elementary Forms of the Religious Life*, Swain's Trans., Chap. 1.

that words "affect us often as strongly as the things they represent, and sometimes much more strongly."[3] Our thoughts about things frequently do affect us both more powerfully than the things themselves and very differently from them: memories of our past lives often touch us more deeply than those pasts touched us when they were being lived. It is natural to commend memory proportionately to its vigour and actuality in reinstating the past and imagination when it most perfectly simulates reality. "But at the present day," wrote Longinus," the word (imagination) is predominantly used in cases where, carried away by enthusiasm and passion, you think you see what you describe and you place it before the eyes of your hearers."[4] Such visualisation, such mental re-creation of sights once seen, seems to be an essential, if not the most essential, duty of imagination and memory. Longinus seems to show some favour to this very natural prepossession. "If you introduce things which are past as present and now taking place," he writes, discussing the historical present, "you will make your story no longer a narration but an actuality." His eye is fixed on actualisation as he notes the visualising effects of questions and interrogation in the oratory of Demosthenes:[6] the rhetorical device of asking questions and giving answers converts description into reality. He seems, however, to perceive a power in imagination superior to the mere photographic representation of reality when he contrasts "enthralment" by the "poetical image" with "vivid description" by the rhetorical.[7] Burke saw very clearly that, as a photograph which simply reproduces with fidelity is not art, so imagination is neither mere visualisation of the real nor to be estimated by its success

[3] *An Essay on the Sublime and Beautiful*, 5-7.
[4] *Longinus On the Sublime*, trans., Rhys Roberts, 15-1.
[5] *Longinus On the Sublime, trans.*, Rhys Roberts, 25-1.
[6] *Ibid*, 18-1.
[7] *Ibid*, 15-8.

in achieving actuality. "To represent an angel in a picture, you can only draw a beautiful young man winged; but what painting can furnish out and anything so grand as the addition of one word, 'the angel of the Lord?'"[8] Imagination's separateness of office from mere photographic reproduction or simulation of reality is recognised throughout his famous essay as it is recognised in this sentence. He recognises it most explicitly where it is most evident—in the use of words. "I am convinced," he writes of "compound abstracts" like "virtue" or "persuasion," "that whatever power they may have on the passions, they do not derive it from any representation raised in the mind of the things for which they stand."[9] "The influence of most things on our passions is not so much from the things themselves as from our opinions concerning them," he adds, and, again, "poetry and rhetoric do not succeed in exact description so well as painting does; their business is to affect rather by sympathy than by imitation; to display rather the effects of things on the mind of the speaker or of others, than to present a clear idea of the things themselves."[10] Burke is analysing aesthetic and poetic effects but it never is the principle or most important function of imagination to achieve a complete mental simulation of reality or of memory to construct a perfect mental duplicate of the past. Art is not mere imitation, neither is memory, nor imagination. These principles are interdependent, if they are not essentially one, for art is more than imitation because imagination and memory, on which it is founded, are, in their essence, more than mere methods of mental duplication.

Visualisation, mental seeing rather than thinking, or mental seeing for the purposes of thinking, is probably

[8] *Essay On the Sublime and Beautiful*, 5-7.
[9] *Ibid*, 5-2.
[10] *Ibid*, 5-4.

more prominent in primitive than in civilised men. "There is reason to believe," writes Rivers, "that sensory imagery is more vivid and more necessary to the savage than to civilised persons, many of whom are able to conduct their lives so as to be indistinguishable from the rest though the power of expressing their thoughts by means of sensory imagery is very defective or even wholly absent."

"A difference in such a subjective character as the vividness of imagery among different peoples is not, of course, a theme on which it is possible to produce direct evidence, but the conclusion that imagery is especially vivid and necessary among savage peoples fully accords with their almost exclusive interest in the concrete, with the high degree of development of their powers of observation, and with the accuracy and fullness of memory of the more concrete events of their lives. This conclusion is supported by observation of their demeanour when describing events they have witnessed. I well remember the first time on which I had the opportunity of observing this. On Murray Island, where I gained my first acquaintance with savage people, courts were held by a British official in collaboration with the native chiefs, at which disputes were settled and offences punished. On the first occasion on which I attended these courts an old woman gave a vigorous and animated account of her experience in relation to the case. As she gave her evidence she looked first in one direction and then in another with a keenness and directness which showed beyond doubt that every detail of the occurrences she was describing was being enacted before her eyes. I have never seen a European show by his or her demeanour with an approach to the behaviour of this old women, how closely knowledge and memory depended on sensory imagery."[11]

After civilisation has made inroads into visualising

[11] *Dreams and Primitive Culture*, pp. 11, 12

power James can say confidently that some people have "practically no visual images,"[12] and a "correspondent" that, in spite of many efforts to acquire visual memory, he must make "certain mental notes while observing the object" to "make an accurate memory drawing" because he has "no image worth mentioning" from which to draw his "image of the object."[13] A legion can be decimated without being completely destroyed and visualising power still persists, though scattered among fewer individuals. Professor Titchener says he can read off a memory manuscript.[14] He and his fellow-visualisers seem to benefit by not suffering forfeiture of their legacy from the past. It is an advantage to read manuscripts from memory; it is also an advantage for a blindfolded chess-player to see board and men in his mind. An American who played chess blindfolded told Taine that, after a fixed look at the board before the game began, he followed every move in his mind as if he had seen it played. He saw board and men at the end of each move as if there were no bandage on his eyes. He was so used to visualising each move as it was told to him that he was actually more easily deceived when he looked at the board than when he pictured it in his mind. Visualising power also helps in arithmetical calculation: Colborn, a young arithmetical prodigy, told Taine that he saw his calculations clearly before him and "another" that he saw the figures as though they were written on a slate.[15].

These feats of visualisation seem to be successes for the real functions of imagination: perfect memory pictures are useful and imagination seems to be dramatic and effective in proportion to its power of presenting pictures—to

[12] *Principles of Psychology: Imagination.*

[13] *Times Educ. Supp.*, Jan. 15, 1920. "What Is Imagination?"

[14] *Lectures On the Experimental Psychology of the Thought-Processes*, Chap. 1.

[15] *De L'Intelligence*, Tome 2, Chap. 1.

its profusion and vividness of authentic imagery. Visualisers like Titchener and Colborn seem to have escaped from a process of decay which civilisation has inserted into primitive mental powers. Writers, like Shakespeare, who abound in images, who are fertile and vigourous in metaphor, seem to rise to their mental heights because visualisation has tarried with them after departing from others. But visualisation has its victims no less than its beneficiaries. Taine cites the experience of a gendarme who had charge of a condemned criminal and saw his head severed by the knife of the guillotine. The terror of that fatal moment settled on his soul. He imagined that his superior officer, suspecting him of guilt, was planning his arrest; voices shouted accusations at him from the clock; he saw the guard approach and soldiers surrounding his house. He armed himself, fled to a wood and resisted all attempts at capture—driven into exile by obsessing visions.[16] Images, pictures in the mind, which are part of thought, are reacted to as if they were things when they copy reality too faithfully. Vivid visualisation, comparable in vigour and clearness of picture to the actual seeing of physical objects, seduces the mind to confuse things with its thoughts about them. A rush of memory, which is thinking about the past, may momentarily persuade us that we are in the past again. Dreams constantly reinstate the past in literal truth through the posing of memory as actual event. Memory defeats its own purpose when the mental image too vigorously simulates reality and imagination, if it visualises right to the top of its bent, compels the mind to behave as if it were among real scenes instead of being surrounded by thoughts.

Visualisation, mental picturing, also induces a systematic, reflective, theoretical confounding of thoughts with things. Yeats quotes Shelley's saying that "thoughts

[16] *De L'Intelligence*, Tome 2, Chap. 1.

which are called real or external objects" differ only in regularity of occurrence from "hallucinations, dreams and ideas of madmen." An expositor might conclude from this to an idealistic identification of all things with thoughts, which is one way of interpreting an essential similarity between visual pictures impressed upon the mind by visible objects and visualised images in hallucinations and dreams. Yeats inverts this idealistic identification into a realistic homology of mental images with physical things: instead of making external objects into thoughts or images he makes images into external things like stocks and stones. "If all our mental images no less than apparitions (and I see no reason to distinguish) are forms existing in the general vehicle of Anima Mundi and mirrored in our particular vehicle, many crooked things are made straight."[17] Visualisers are always tempted into this psychological or metaphysical delinquency of conferring the externality appropriate to physical things upon mental images. "Forms existing in the general vehicle of Anima Mundi" have obviously received this conference. Images are mental devices for thinking about things (and also about other entities). A writer can invite thought about horses by using the word "horse": the word received as a picture by his readers does not induce them to confuse thinking about the animal with a belief that it stands before them. He can extend the same invitation by drawing a picture of a horse and his readers can again think about horses without supposing them to be present in person. The mental picture or image of a horse, as distinct from a perception of a real, present animal, serves the same purpose for thinking as the writer's picture: it is not the animal, or, in essence, an attempt to portray it as if it were present, it is a mental instrument for thinking about it. The mental visual image could not be adequately replaced by

[17] *Per Amica Silentia Lunae*, Chap. 7.

the word "horse" if it were in essence an imaginative attempt to portray the animal and not essentially a means of thinking about it. But the visual images involved in visualisation, because they are instruments of thought based on simulation of the visual aspects of things which convey to us our most impressive and most systematised experiences of outer realities, if imagination centres on them too freely, become confused with external things. They hallucinate if imagination approaches too closely to a positive reproduction of a perceived reality; they promote systematic mental confusion when imagination centres too exclusively upon them as objects, rather than as devices for thinking, without actually conferring upon them the perceptive status of external things. Yeats has obviously succombed to this impulsion into mental confusion. "A. E." also succombs, and unreservedly, to the visualiser's impulse when he writes "But the dream figure or figure of imagination will walk about with authentic motions and indistorted anatomies."

"I imagine," he continues, "a group of white-robed Arabs standing on a sandy hillock, and they seem of such a noble dignity that I desire to paint them. . . I say to myself, 'I wish they would raise their arms above their heads,' and at the suggestion all the figures in my vision raise their hands as if in salutation of the dawn. . . My brain does not by any swift action foresee in detail the pictorial consequences involved by the lifting of arms but yet by a single wish, a simple mental suggestion, the intricate changes are made in the figures of imagination as they would be if real Arabs stood before me and raised their hands at my call."[18] He succombs to the visualiser's impulse because he concludes from experiences like this that images can enter the mind from *without*. When "A.E." subsequently writes "I remember incidents rather

[18] *The Candle of Vision:* "Have Imaginations Body?"

than moods, vision more than ecstasy"[19] he probably intimates his mental habit of attending more to his visualised images than to their significance: of accepting them as models of realities rather than as means of thinking about these realities. He habitually treats them as models by painting from imagination in preference to sight. The visualising habit, convenient for specific reference in memory, converts the mental image from its essential rôle as an instrument of thought into an actual duplicate or model of external things. The model becomes confused with its prototypes in the outer world and "A. E." can write, treating the mental image as a ball that can be thrown from hand to hand, "I know that with the pictures of memory mingle pictures which come to us, sometimes from the minds of others."[20] He can speak of the "etherial medium which is the keeper of such images," thus relegating mental images to the outer existence and persistence of pebbles, of his ability "at times to evoke deliberately out of the memory of nature pictures of persons or things long past in time"[21] and even suggest "that images of things to be may come into our sphere out of a being where time does not exist."[22] Visualisation has its conveniences and it has its powers, but the essential function of imagination, in all its forms, is not picturing but the use of images as instruments of thought.

Human thought often seems to follow a route decided for it beforehand by the constitution of things and by the equipment of the mind. It was probably inevitable for Plato to start in one such route and write "Then must we not infer that all the poets, beginning with Homer, are only imitators; . . . The poet is like a painter who, as has

[19] *Ibid:* "Intuition."

[20] *The Candle of Vision:* "The Mingling of Natures."

[21] *Ibid:* "The Memory of Earth."

[22] *Ibid: "The Architecture of Dream."*

already been observed, will make a likeness of a cobbler;"[23] and "that poetry and mythology are, in some cases, wholly imitative."[24] Plato's thought was not tied to a narrow identification of art, even of most prominently representative art like painting, with imitation; but it was very obviously determined by the conception, so naturally adopted in the first instance, of art as skillful copying. Hannay observes that primitive art, like the art of children, attempts to put a line round a mental conception more than it endeavours to represent what the eye perceives.[25] But it is always difficult to understand what we really do and it was probably inevitable that aesthetic theory should begin by comparing painting, and even poetry, to copying or imitation. With an equal inevitability aesthetic theory could not persist in this inadequate description of art: Plato's own thought constantly burst through its confining bonds. The core of truth in the imitative theory of art, for artistic expression does employ the representative element of imitation, does, however, continually maintain a sympathy with Burke's dictum:" in the imagination, besides the pain or pleasure arising from the properties of the natural object, a pleasure is received from the resemblance which the imitation has to the original: the imagination, I conceive, can have no pleasure but what results from one or other of these causes."[26] Explorative thought, however, because it is explorative, finally recedes from imitation as the constitutive element in art. "Why are we not always pleased," asked Sir Joshua Reynolds, "with the most absolute possible resemblance of an imitation to its original object? Cases may exist in which such a resemblance may be even disagreeable. I shall only observe that the effect of figures in wax-work, though certainly a more exact

[23] Jowett's Trans., *Rep.* 10, 600, 601.
[24] Jowett's Trans., *Rep.* 3, 394.
[25] "Photography and Art": *The London Mercury*, Jan., 1920.
[26] *Essay On the Sublime and Beautiful*, Introd.

representation than can be given by painting or sculpture, is a sufficient proof that the pleasure we receive from imitation is not increased in proportion as it approaches to minute and detailed reality; we are pleased, on the contrary, by seeing ends accomplished by seemingly inadequate means."[27] Sir Joshua's discourses definitely transcend the identification of art with imitation. "If we suppose a view of nature represented with all the truth of the camera obscura, and the same scene represented by a great artist, how little and how mean will the one appear in comparison with the other, where no superiority is supposed from the choice of subject. The scene shall be the same, the difference only will be in the manner in which it is represented to the eye. With what additional superiority then will the same artist appear when he has the power of selecting his materials as well as elevating his style?"[28]

Mandeville, some fifty years before, still described painting as an imitation of nature, as a "happy deceit" possible through imperfections of sense; and ingeniously, though perversely, suggested that reflections from polished bodies were the first cues to the invention of painting.[29] The seduction by the element of counterpart in drawing, sketching, sculpturing or narration, by the representative substratum in art, into identifying art with imitation is hard to resist and difficult to expel. Fidelity in portraiture is still a popular criterion of a picture and it was for long a canon of more reflective opinion. The simulation by the visual mental image, by the mental picture, of outer things, analagous to the simulation by a painting of the scene it represents, has seduced thought into psychological and philosophical conceptions analagously to the imitative theory of art. The identification of artistic conception with

[27] Eleventh Discourse.

[28] Thirteenth Discourse.

[29] *A Search into the Nature of Society.*

the picture, with the painted image, with the representative sculptured form, with the imitative element in description or narrative, has a natural parallel in the identification of thought with mental image. No one of these indentifications has ever been absolute: reflective minds cannot constantly and consistently regard all artistic conceptions solely as imitations, neither can they continuously suppose an inclusion of all thought in mental imagery. But in both instances a fundamental, dominant prepossession has perpetually infected thinking: artistic conception, on the one hand, has been constantly confused with the picture which embodies it, and the mental image, on the other, has been systematically confounded with the process of thinking it subserves.

The mental image often has a valuable transcriptive function: visualising memory is a mental convenience for the blind-folded chess-player; pictures have frequently a photographic value for those who have lived among the scenes they represent or who desire to realise those scenes; and Burke rightly says that "a pleasure is received from the resemblance to the original." So also it is pleasant simply to have mental pictures of the past. Mere imitation has its pleasures and it has its uses. But exclusive preoccupation with the mental imitative picture or visual image, like persistent centering on the imitative in art, leads to confusion, mystification and error. Some modern tenders of the fires of art protest against the prerogative of imitation by destroying all semblance, in their pictures, to anything in earth or heaven. The mind has done with its imagery, in its own way, what these enthusiasts have done with their pictures. Let any one who is not habitually or preeminently a visualiser, perhaps even then, recover by memory and inspect with attention his mental imagery during a process of thought. He will not usually discover a visualised duplicate of any outward scenes nor an

ordered, mentally imaged version intelligibly reconstructed from such scenes. His imagery, simply as a collection of images, will be a kaleidoscopic chaos. The images themselves will normally be sketchy, shadowy, vague and undefined—mere ghosts of their physical prototypes. The following passage from Burke well expresses the disintegrated spread of imagery through most thought—dissolved in the mental process like salt in water. "If I say, 'I shall go to Italy next summer' I am well understood. Yet I believe nobody has by this painted in his imagination the exact figure of the speaker passing by land or by water, or both; sometimes on horse-back, sometimes in a carriage; with all the particulars of the journey . . . it is not only of those ideas which are commonly called abstract, and of which no image at all can be formed, but even of particular, real beings, that we converse without having any idea of them excited in the imagination; as will appear on a diligent examination of our minds."[30]

It is probable that many dreams essentially raise up before consciousness, into a more vivid realisation, the disbursed and depressed imagery involved in a sequence of thought. Titchener tells us that he habitually receives, when thinking of modesty, a "visual hint" in a visual image of a graceful, bending female form.[31] If he thought of one of his students, "Mr. Smith should be more modest," his mind might, during the night, confer upon the mental images involved the perceptive status of a dream. Mr. Smith would appear in the dream in person, since the mental picture of his person would carry Titchener's thoughts about him; modesty would appear in the guise of its "visual hint"—a modest young lady. Thus a dream of a somewhat sentimental meeting might result from a prominence in consciousness given to the imagery of a

[30] *Essay On the Sublime and Beautiful*, 5, 5.

[31] *Lectures On the Experimental Psychology of the Thought-Processes*, Chap. 1.

thought. The illusion inherent in the dream would depend upon diverting the mental images from their proper function as devices for thinking to imitations of real events. If dreams do so arise, whether at times or always, they intimate to us, by our false reactions to relatively vivid images as if they were realities, how shadowy our normal imagery is, and frequently, by their wandering or by their eccentric combinations, how chaotic is the imaginative part of thought. Rivers thinks the visualising habit is stronger among primitive and more degenerate among civilised peoples. The pulverising of mental imagery as development proceeds warns us off false estimates of the image, as modern chaoses in paint seek to divert mistaken emphasis on the imitative in art.

The tendency to consider mental imagery as essentially imitative, as a collection of photographs, instead of realising that the mind more resembles, in Titchener's phrase, a picture gallery of impressionist notes, long confused discussions on General Ideas. We certainly have general ideas for we can speak or think of "man" in a general sense and "animal" is as intelligible with a universal as with a particular significance. The capable thinkers of the past were not entirely in the dark but their mental vision was obscured by a confusion between "idea" or "thought" and "mental visual image." A mental picture of a horse is individual or particular; it may, however, assist the mind to think about horses in general: it may *refer* to all horses though it only portray one. Mental imagery, visualisation, before it produced this perplexity, apparently had a previous effect of hindering any formation of general ideas at all: before puzzling philosophers about the nature of general ideas it prevented them from acquiring them. Many observers have noted that primitive people tend to identify a drawing or picture of any animal with a particular individual. Wernlé says, for example, that East African

natives always draw particular crocodiles, never representing crocodiles in general by a picture of one. An English child soon understands that a picture of a canary in its animal book represents many canaries; the East African identifies a picture of a crocodile with the one he saw yesterday or with the one which ate his grandfather.[32] Visual mental images tend to persuade the mind, as pictures of crocodiles persuade East African natives, of the presence of some particular things or scenes. Dreams indulge this tendency by convincing the dreamer that he is in a boat or pursued by a tiger or falling through a window. The natural mental tendency, insistent in primitive identification of pictures of crocodiles with special crocodiles and emphatic in persistent comparison of the visual mental image with an attempt to reproduce reality completely, to regard all representations merely as reproductive facsimiles resulted naturally in supposing art to be imitation and in erroneous notions about the function and nature of mental imagery.

Berkeley could realise that a picture of a crocodile need not refer to the particular crocodile that he happened to have seen the day before but he had difficulty in understanding *how* an individual mental image need not represent only a single individual. "The idea of a man that I frame to myself," he writes, "must be either of a white, or a black, or a tawny, a straight, or a crooked, a tall, or a low, or a middle-sized man, I cannot by any effort of thought conceive the abstract idea of man."[33] Hobbes also considered it erroneous to speak of the idea of anything as universal because there cannot be in the mind an image of a man which is not the image of some one man.[34] The fascination of the mental image, its obstrusiveness, per-

[32] *Native Life in East Africa.*

[33] *Principles of Human Knowledge,* Intro. 10.

[34] *Elements of Philosophy,* 1-3.

sistently induced mental confusion by persuading men to identify imagery with thinking. In a parallel fallacy art was supposed to be imitation by mistakenly identifying *picture* with conception.

A day by the sea-shore is pictured, five years after, in memory. No particular emotion prominently connected itself, at the time, with the events of that day. The time was filled with action—with walking, bathing, conversation and the detail of life's round. No action is invited by the remembrance, which, in place of it, instals the reminiscent mood. The peculiar emotion characteristic of reflective reminiscence, springing from the memory of past scenes, differs from the feelings excited when those past scenes were real. Living through an event and remembering it are two different experiences and have their own separate emotional accompaniments. If memory literally reinstated the past it would restore a moment of time to its original being. But the function of mental imagery, even in memory, where duplication of the past event is most complete, is not to imitate but to present for thought. The more perfectly imitation is secured, for mental imagery often alludes to its theme by employing an element of imitation, the less perfectly is imagination's office discharged. Insistently imitative or reduplicative images hallucinate and destroy rational thought by falsely simulating reality, as dreams delude us into mistaking the imaginary for the real. "The truth is, if poetry gives us a noble assemblage of words corresponding to many noble ideas which are connected by circumstances of time or place, or related to each other as cause and effect, or associated in any natural way, they may be moulded together in any form, and perfectly answer their end. The picturesque connection is not demanded; because no real picture is formed;"[35] it took longer to perceive that what Burke here realises to be true

[35] *Essay On the Sublime and Beautiful*, 5-5.

of poetry may also be true of painting *viz.* there need be no "picturesque connection," no imitation. Art is not mere imitation, nor necessarily imitation at all, for the same reason that mere visualisation, simple portrayal, complete duplication, is not the essential function of mental imagery. Mental images resemble real objects, when they discharge their unperverted or primary function, only sufficiently to direct the mind to the themes they subserve—only sufficiently to secure adequate allusion. They can allude adequately without being directly representative at all: words, which can appear as mental visual images, have no claim to be considered imitative. Accurate visualisation has subsidiary uses: reading off memory manuscripts is a convenience and accurate reproduction in memory may recover significant fact. But mental imagery has for its chief use the impressing of past experiences into the service of thought.

The possibility of an impressiveness in the symbol superior to the impressiveness of the thing symbolised is obviously inherent in the essential function of mental imagery. The memory picture of a past event is a symbol of that event. The emotional reaction to the reminiscent symbol may be greater, and often is greater, than to the event symbolised because the symbol does not require the practical reaction, the actual living towards, demanded by the reality. An external symbol may resemble the mental image in freely admitting emotional reaction by suppressing the need for the customary habitual reactions of life.

Art takes advantage of this principle. Imitation, pure and simple, would tie it to a poor repetition of the actual. By transcending the imitative art secures another value which may be richer than the value of actuality itself.

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CRITICISMS AND DISCUSSIONS.

ON THE LOGICAL INTRODUCTION TO THE THEORY OF FUNCTIONS.[1]

It seems as though it would still be premature to pronounce definite judgment on the results obtained by logistics, to attempt immediately to draw up, as it were, its scientific balance-sheet. Many questions that come under the logistical doctrine have not yet been solved, and one of the most important works by Whitehead and Russell, a work which must contribute to the establishment of this branch of science, has been announced, though it has not yet been published.[2] Notwithstanding this, it may not be without interest to recapitulate certain points in the discussion which has recently been raised along these lines between mathematicians and logicians, and to attempt to deduce some conclusions therefrom.

This controversy had a metaphysical point of departure. The first thing was to find out what element of intuition and of logic there was in mathematical reasoning. Kant regards intuition (*Anschauung*) as a mode of thinking which involves sensibility; in his philosophy, the determination of the nature of this intuition is of supreme importance, since, in his system, analysis of the elements of knowledge is essential. The investigation, however, which aims at determining the precise characteristics of intuition and the relations of this intuition to thought in general, constitutes a transcendental speculation, which could not be the real object of debate, the theory of intuitionistic metaphysics being no more susceptible of proof than that of the intellectualistic metaphysics of logisticians. In these matters, the argumentations of the most learned mathematician have no greater value than the discourse of the most ordinary philosopher,

[1] [Authorized translation by Fred Rothwell from "Sur l'Introduction logique á la théorie des fonctions," *Revue de Métaphysique et de Morale*, Vol. XV, 1907, pp. 186-216.]

[2] [Whitehead and Russell's *Principia Mathematica* began to be published in 1910 and is not yet complete.]

for the very reason that the mathematician no longer thinks as a geometrician but as a metaphysician. The problem, stated in scholastic terms, admits of no solution.

Nevertheless, what seemed to constitute the interesting element in this philosophical debate, was that the existence of a scientific theory, or at all events of a doctrine which offers a positive character, appeared to be connected with its solution. If intuitionism is the true doctrine, the works of Boole, Schröder, Frege, Peano, Whitehead and Russell must disappear; if logical intellectualism expresses true philosophy, the works of the logisticians are justified. Well then, in our opinion there is no solidarity at all between the metaphysical controversy, to which we have just alluded, and the question as to whether this special science called logistics has a solid foundation or not: the existence of logistics no more depends on controversies as to the nature of mind than the legitimacy of experimental physics is connected with discussions as to the inmost essence of matter. No doubt certain logisticians have attributed excessive importance to their own science, and, in a sense, have transformed it into scholasticism; this metascientific extension of a positive doctrine is illegitimate and raises insoluble antinomies. But we believe there to be a scientific and restricted conception of logistics, which is free from the fluctuations of philosophic thought: we will attempt to prove this theory shortly. Before entering into this essential point, we must remember that the constitution of logistics is in conformity with the meaning of the evolution of the sciences, and, without entering into many historical details, a few brief indications will enable us to justify this affirmation.

It is known that the works of Cauchy and Abel are at the root of the development of analysis in the nineteenth century, and that logical precision and rigorous demonstration characterize the work of these great geometricians. In 1826, Abel wrote to Hansteen: "I shall devote all in my power to bring light into the vast darkness now reigning in analysis.... In higher analysis, very few propositions are demonstrated with strict rigidity. Everywhere we find that unfortunate method of inferring from the special to the general....[3] This need of strict demonstration corresponded to logical requirements, to the necessity of reducing as far as possible the approximations of the imagination of inventors. Cauchy and Abel have contributed largely to the acceptance of the principle—now undisputed

[3] Abel, *Œuvres*, Sylow and Lies edition, II, 263.

—that no propositions in mathematics can be accepted unless it has been fully demonstrated. The geometricians of the eighteenth century, however, less exacting than modern mathematicians, were satisfied with methods the exactness of which left much to be desired, since for the most part they trusted to their intuition.[4] Suffice it if we recall the erroneous conception of divergent series, of Euler's series, for instance,

$$1-1+1-1\ldots.$$

As the sum of this series is alternately 1 and 0, according as we take an odd or an even number of terms, the geometricians of the eighteenth century regarded it as natural to take the arithmetical mean of 1 and 0, i. e., ½, as the sum of the series. This reasoning, however, is inexact, "for it is easy to form series the terms of which depend on a variable, and which reduce to Euler's series for a particular value of this variable, in such a way that the limiting value of the series is absolutely any number whatsoever."[5] The geometricians of the eighteenth century employed divergent series because these series were successful empirically; they did not trouble about the logical difficulties which the series might raise. What we have just said regarding divergent series would be equally correct regarding many other branches of mathematics. Here again we would refer to the transformations in the definition of the integral. Analysts, from the time of Cauchy and Dirichlet, have endeavored to substitute, for the confused notion of the integral considered as an area, a rigorous analytical definition. Riemann's works throw light on the matter, whilst the results obtained by Henri Lebesgue at the present time must be mentioned. Here too, exact analytical definitions are substituted for vague intuition. We may therefore say, confining ourselves strictly to historical truth, that the analysis of the eighteenth century was far less rigorous than that of the nineteenth, and, unless we assert that the results of the nineteenth century are inferior to those of the eighteenth, we must acknowledge that the entire evolution of mathematical science points to progress along the lines of a greater logical precision.

The affirmation of Pierre Boutroux that "it is frequently advantageous, in the theory of functions, to substitute for the study

[4] We use the word "intuition" in its ordinary meaning. Intuition is a *rapprochement* made by the imagination, a *rapprochement* that is not tested by logical criteria.

[5] Borel, *Leçons sur les séries divergentes*, p. 6.

of a development that of less precise and more *intuitive* characters,"[6] must be understood as meaning that, in research, it is convenient to employ less rigorous methods; though unquestionably the results obtained, before being definitely incorporated in science, must be verified by the most exact methods. To assign any other meaning to this affirmation would be to deny the entire development of modern mathematics.

Ever since the progress of mathematical methods consisted in attributing an ever greater part of logical rigor and precision, we had to examine the very foundations of the reasonings of geometricians, to analyze the fundamental types of demonstrations, and to establish the indefinable elements on which these demonstrations are based. Logistics, therefore, was in conformity with the spirit of the works of Cauchy and Abel. Still, though the historical development of mathematics certainly seems to prove that logistics is legitimate, *de jure*, at all events, how far can it be said that the logical reduction of mathematical reasonings is accomplished *de facto?* Do the nine indefinable and the twenty indemonstrable propositions of Russell constitute the whole logical framework of mathematics? These are questions to be examined in detail. It seems that there are problems which Russell has solved and others in which he has failed; it would scarcely be reasonable to expect him, unaided, to build up the whole of logistics. Note, moreover, that Russell would appear to be mistaken as to the import of logistics when he says: "The fact that all mathematics is symbolic logic is one of the greatest discoveries of our age."[7] It seems, however, that he himself restores logistics to its true import, when he regards it as a sort of speciality,[8] a branch of mathematics, unknown indeed to the majority of mathematicians, and not the indispensable instrument of all mathematical investigation. What are the exast limits of this new branch of mathematics? What is its positive function, and what are its relations to the great mathematical theories, especially the theory of functions? These are the questions that have to be answered. They are weighty problems, however, and we do not pretend to solve them in the following pages. We will confine ourselves to emphasizing certain points.

[6] Pierre Boutroux, *Thése sur quelques propriétés des fonctions entiéres*, p. 1.

[7] Russell, *Principles of Mathematics*, 1903, p. 5.

[8] Russell, *Revue de Métaphysique et de Morale*, Sept. 1906, p. 628.

I. THE GROUND COVERED BY LOGISTICS.

We will at the outset attempt to give a few indications as to the determination of the limits of the domain of logistics. It must have boundaries in the sense of metaphysical abstraction (a upper limit if you like), signifying that it must, by an explicitly formulated postulate, put a stop to all digressions in the barren domain of scholasticism.[9] It must likewise have boundaries as regards its applications (lower limits), or, if you will, its extension to the domain of positive sciences, in order to avoid clashing with existing methods. We shall take for our starting point Whitehead's definition of a calculus in general. A calculus is defined as "the art of manipulation of substitutive signs according to fixed rules, and of the deduction therefrom of true propositions.[10] According to this meaning, still a very vague one, a calculus may apply to two kinds of mathematical entities: *non-numerical* and *numerical.* These two kinds of entities differ in the fact that the former verify the following laws:

$$a=aa$$
$$a=a+a$$
$$a+ab=a;$$

whereas the latter satisfy the well-known rules:

$$aa=a$$
$$a+a=2a$$

(we give these rules only in order to state exactly what we think, we do not mean that the elementary laws governing the operations of ordinary arithmetic can be applied to all numerical mathematical entities).

The field of a calculus, comprising both numerical and non-numerical entities, contains all possible forms of thought: according to this general sense, then, logistics is a calculus.

But with regard to a calculus in general, there may first be asked the necessary philosophical question: what relation does reality hold to rules and to substitutive symbols? In our opinion, a problem thus stated has no scientific meaning, hence, whenever problems of this kind are asked, we may neglect them. In doing so, however, by a tacit postulate we confer absolute value on the logico-mathematical laws, we apply to them the idealistic principle (the laws of mind are

[9] Those who regard this precaution as unnecessary have only to read the articles on this subject in the *Revue de Philosophie.*

[10] Whitehead, *Universal Algebra,* p. 4.

the laws of things) which others apply to metaphysical principles and we thus put a stop to all wandering from the point in the field of scholasticism, as well as to questions which admit of no reply, such as the following: what is the objective value of symbolism? We should not have dwelt so long on this matter, had not Louis Couturat, an authority in questions of logic, affirmed that speculative philosophy comprises "in addition to the methodology of the sciences, the epistemology or criticism of the principles of the sciences, the general theory of knowledge, and finally metaphysics as the science of being, so far at least as this latter is known and knowable, and is conceived in its relations to the mind.[11] Now, if the methodology and the epistemology of the sciences are legitimate, it appears to us erroneous to affirm the justification of a theory of knowledge and of a metaphysic. Having already explained our position on this point, we will not dwell on it any longer.[12]

It remains for us to determine the lower limit of the field of application of logistics, a far more delicate task, for it does not appear as though we could yet obtain any definite results from the controversy on the subject. The problem is stated in the following general terms: apart from a certain domain proper to it, which Hadamard defines as that of the analysis of mathematical reasonings[13] and which we shall presently attempt to determine a little more explicitly, should logistics extend, as a new general method, to the different parts of mathematics? Is there a time when the logistician must give way to the mathematician or can they both travel together? Is logistics, in every respect, a *substitute* for mathematics? If Peano has not fully adopted the latter alternative, he at least seems to regard logistics as a new algebra. "He uses it," says Jules Richard, "*as algebra is used* for stating propositions and deducing them from one another according to fixed rules. As I could prove by numerous quotations, he has never thought of making up for the primary notions of science, that of number, for instance, by this algebra."[14] But does not this mode of regarding logistics imply an illegitimate extension of its field of applicaton? The argument that merely says that the *Formulaire* exists,[15] i. e., that it is in print, is of no great importance. The request to the opponents of logistics to find errors

[11] Couturat, *Revue de Métaphysique et de Morale*, May 1906, p. 340.
[12] *Revue de Métaphysique et de Morale*, July, 1905.
[13] Hadamard, *Revue générale des Sciences*, 30 Oct., 1906, p. 909.
[14] J. Richard, Ibid., 30 Nov. 1916, p. 957.
[15] Couturat, *Revue de Métaphysique et de Morale*, March 1906, 220.

of demonstration in the *Formulaire*[16] is certainly more serious, though it does not remove the main objection. If logistics, when it takes up the work of the mathematician himself, is but a *translation* into other signs of ordinary mathematical reasoning, as it is accused of being, then it can make mistakes only if the original reasoning has made mistakes, in this case, logistics is blamed not for its falsity but for its *uselessness*: a method having justified its existence either when it permits of the solution of problems not previously solved, or when it is more precise and rapid than existing methods. The logistical method has not provided a solution of problems strictly mathematical and not previously solved, nor can it lay claim to rapidity; so that it remains to be seen whether, in the strictly mathematical domain, not only in the domain of the principles of mathematics, it possesses qualities of exactness which the standard methods do not possess, by explicitly formulating all the postulates which are mostly used unconsciously.

As a matter of fact, say the logisticians, the reason why the mathematician is not mistaken is because he is guided by unconscious logic. Let us see, then, in the most elementary of examples,—the arithmetical addition of integers,—if there is room for the intervention of logistics in the strictly scientific domain. We will leave out of account the questions on principles (definition of number, etc.), since, from our point of view, these come under the heading of logistics.

The operation $+$ will be characterized, as in all treatises on arithmetic, as a primitive operation having the four properties:

1° Of being associative;

2° Of being commutative;

3° Of having zero for its modulus;

4° Of being such that, if to a number a we add numbers differing from one another, we still obtain numbers differing from one another.

Everyone knows that these properties were formulated long before the existence of logistics. Suppose the system of numeration determined, as well as the rules for writing numbers in the chosen system (we refer to treatises on arithmetic for details of three rules) and that we have to add together $5+7+(3+4)=19$. Will the rules established by arithmetic enable us to avoid all error? Are mistakes avoided by virtue of an unconscious logic or by virtue of the rules

[16] *Ibid.*, p. 221.

of arithmetic? In a word, does not arithmetic itself use logic? And is not all error in calculations of addition impossible when we correctly apply the rules of arithmetic? Is not the intervention of a logical calculation, in verifying the correctness of the operation, supererogatory?

We do not think there can be any doubt on the matter. If our example is regarded as too simple, the same question may be asked in the case of trigonometrical calculation. At the outset, we may introduce the trigonometrical functions sine x and cosine x in the way indicated by Jules Tannery so as to reduce experimental data as much as possible: "We also establish from geometrical considerations the formulas:

$$\alpha \begin{cases} \sin\ (a+b)=\sin\ a\ \cos\ b+\cos\ a\ \sin\ b. \\ \cos\ (a+b)=\cos\ a\ \cos\ b-\sin\ a\ \sin\ b \end{cases}$$

I will show (taking their existence for granted) how we may determine all the continuous functions $\phi(x)$, $\psi(x)$ which have preperties defined by the formulae:

$$\beta \begin{cases} \phi(a+b)=\phi(a)\phi(b)-\psi(a)\psi(b) \\ \psi(a+b)=\psi(a)\phi(b)+\psi(b)\phi(a) \end{cases}$$

and likewise satisfy another condition.... Designating by cos x and sin x functions which are known to be continuous and which must satisfy the equations (α)....we must have the relation

$$(\gamma)\ \sin^2 x+\cos^2 x=1" \quad [17]$$

The functions sin x and cos x are determined by formula (α) and (γ). Considering the functions sin x and cos x as given by their developments in series; it may be shown how these functions effectively satisfy the conditions with which we started. Without taking up the demonstrations in every detail, we see that here is a starting point for a rigorous establishment of the formula of elementary trigonometry.[18] By strictly keeping to the rules thus obtained, we may be certain that we shall never make mistakes in any particular trigonometrical calculation. It will not be unconscious logic, but a literal application of the formula that will have enabled us to avoid mistakes. Moreover, as all the reasonings of algebra, analysis and geometry are as rigorous as those just mentioned, and are all ob-

[17] J. Tannery, *Introduction á l'étude des fonctions d'une variable*, First edition, p. 146.

[18] *Ibid.*, p. 157.

tained by the ordinary rules of calculation, it is not clear what useful a part logistics is capable of playing within the particular domain of these different sciences.

Perhaps Peano and his collaborators had the impression that, by confining themselves to the determination of the grammatico-logical principles of mathematics, logistics would constitute no more than a limited speciality, and as the idea of a new general method was probably continually in their minds, they have included in the *Formulaire* a number of pages which, strictly speaking, ought not to be there. As an instance, amongst many others which might be mentioned, we will refer to pages 35, 40 and 41 in volume V of the Formulario matematico....

Speaking generally, we do not see what practical algebraical calculation is to gain from these evident remarks. From the beginning of algebraical calculation, strictly so called, *the rules of this calculation have become the logical rules*, and no further search is required. To teach the first elements of calculation, however, phrases have had to be employed; indeed, it would be impossible to explain the beginning of mathematics without a preliminary grammatical discourse. "The study of grammar," writes Russell, "is in my opinion capable of throwing far more light on philosophical questions than philosophers generally suppose."[19] Thus we meet with Peano's signs for implication, conjunction, belonging to a class, etc. These signs are substitutive for fundamental ideas. How are we to characterize these ideas, how employ them, and how do they relate to mathematics, properly so called? These are questions to which a reply must be given and which must be treated with the same precision as mathematics itself, if we would prevent the beginnings of arithmetic, analysis and the other branches of mathematics from being lost in mist and fog.

Russell has already thrown considerable light on these questions in Chapter V of the *Principles of Mathematics* which deals with denoting. He brings forward the part played by the words *all, every, any, a, some, the.*[20] As we cannot imagine the state of a man's mind from which all grammatico-logical notions were banished, the very principle of all positive philosophy compels us to study these notions as given facts, to set up the laws of the combinations proper to them, as well as the relations which unite them to mathematical science

[19] Russell, *Principles of Mathematics*, p. 42.

[20] There has since appeared an article by Russell in *Mind* (Oct. 1905) on the question of denoting.

strictly so called. Such is the positive field for the application of logistics, a field which constitutes the logical introduction to the theory of numbers and to the theory of functions.

If we agree with Hadamard[21] that the problem which consists in absolutely reducing the mathematical principle to the grammatico-logical one, or vice versa, constitutes a metaphysical problem that has no possible solution, there would still have to be set up positive and exact correspondences between the notions of both orders. Even admitting, though it is not so, that Aristotle formulated for all time the laws of logic, without the possibility of either adding to or removing anything from its formulas—the theory of Poincaré, which the illustrious mathematician himself destroys by acknowledging that the notion of propositional function constitutes a "happy invention,"[22]—we should still have to determine the correspondence between syllogistic and grammatical notions, on the one hand, and mathematical calculation, on the other. For instance, Whitehead has plainly shown the range and meaning of syllogistic by exhibiting it in its symbolical form; it becomes a particular instance of the general methods of elimination: "It is evident that each syllogism is simply a problem of elimination of the middle term."[23]

The precise determination of the grammatico-logical elements which come into mathematics responds to that need for strictness which impelled Abel to take up again the demonstrations of the theorems of the higher analysis which he regarded as inadequate. This work, however, may also have another effect, that of improving the instrument of non-numerical logic, and of rationalizing grammatical forms by comparing them, and as far as possible reducing them, to mathematical forms.

Consequently, the analysis of mathematical reasonings, i. e., the determination of the grammatico-logical types they contain, constitutes the distinctive domain of logistics. It must be admitted that the determination of the nature and range of the principle of complete induction, for instance, should be dealt with logistically, and that no other way of attacking the problem is scientific. In metaphysical terms, Poincaré states the following problem: How, by an analytical method (conceived as based solely on the principle of identity), can *new* truths be discovered?[24] Poincaré finds the solu-

[21] Hadamard, *Revue générale des Sciences*, 30 Oct. 1906, p. 908.
[22] Poincaré, *Revue de Métaphysique et de Morale*, Nov. 1905, p. 827.
[23] Whitehead, *Universal Algebra*, p. 103.
[24] Poincaré *La Science et l'Hypothése*, p. 10.

tion of the question in complete induction. Complete induction, then, which solves a philosophical problem, is thereby promoted to the dignity of a method-type for mathematics. It is possible, however, that the metaphysical problem originally stated may not be legitimate, scientifically speaking at all events. First, is it true that the analytical method is reduced to the application of the principle of identity? Couturat has refuted this purely tautological conception of the analytical method. Moreover, does the novelty of the discovery for which Poincaré wishes to account, constitute a psychological manifestation with reference to man, or does it correspond objectively to something? If we reveal the laws of nature, that means that they are new *to us*, just as America was new to Christopher Columbus in spite of the previous existence for centuries of the American continent. And is it not manifestly contradictory to think of attributing an absolute logical foundation to what is only psychological and human? Without advancing further into the scholastic labyrinth, these remarks suffice to show that the terms of the problem were badly determined: consequently, any solution of them will be disputable. Now, it is because complete induction solved a metaphysical problem in the mind of Poincaré that he has attributed to it a universal value which it cannot have. Subsequently, Poincaré mitigated the too absolute element in his original conception: "I saw in it the mathematical reasoning *par excellence*. I did not mean, as has been thought, that all mathematical reasonings could be reduced to an application of this principle."[25] But is it a fact that this is mathematical reasoning *par excellence?* As Poincaré's opponents, the logisticians, did not start with a general philosophical problem, and it must be recognized, that their point of departure, at all events, is in conformity with the positive method: they have studied in succession all the forms of mathematical reasoning, and, analysing the first logical notions, have found the principle of complete induction in the theory of *finite* numbers;[26] they would appear to have quite recognized the part this principle plays, regarding it as a fundamental element in the definition of these numbers. Now, are there other forms of demonstration foreign to the principle of induction (as, for instance, in the syllogism $a=b$, $b=c$, therefore $a=c$) and which are in no way applications of this principle? On this point, logistics has determined a certain number of types of reason-

[25] Poincaré, *Revue de Métaphysique et de Morale*, Nov. 1905, p. 818.

[26] Couturat, *Les principles des mathématiques*, p. 62.

ings (the principle of composition, of simplification, of contraposition, of exportation, of importation, of deduction, of substitution, principles of the logic of relations....) the inanity of which would have to be shown before we could affirm the supremacy of the principle of complete induction. Moreover, it is clear that, if we give the name of complete induction to the faculty possessed by the human mind of forming general judgments, valid for an infinite number of cases, such a principle intervenes in every act of thought. There is nothing legitimate about this denomination however. It would appear that the method of complete induction strictly speaking, must only be used in its own proper domain (the theory of finite numbers) and in the cases related more or less directly to this domain. It appears, then, that the logisticians must finally win as regards the principle of complete induction, the problem being distinctly within their province, since we have to analyze a mathematical reasoning.

To sum up, however sarcastically logistics be regarded, it is a positive domain of investigation, an ensemble of questions to which answers must be given. Still, while we cannot ignore this branch of science, it would be equally erroneous to imagine that the essential of mathematical thought has been absorbed in the general theories of logistics and to think that all the other branches of mathematics are reduced to an automatic application of the rules laid down in the logical introduction. Even admitting, as we do, the justification of logistics, though circumscribing its domain very precisely, the absolute autonomy of mathematical thought must be acknowledged. What we mean is that, even if the general questions of logic were solved to the satisfaction of all, the difficulty of the real problems, which we find in pure mathematics or which the physicist asks the calculator, would be in no way lessened thereby. To parody a famous motto, we might say: Logic is well established, now begins the era of scientific difficulties.

In the two following paragraphs we will endeavor to set forth a few examples of problems to the solution of which logistics brings no light or help of any kind; thus we shall determine more positively what we have called its lower limit.

II. ON THE DEFINITION OF IRRATIONAL NUMBER AND ON THE GENERALIZATION OF NUMBER.

In one article, we could not dream of examining in succession all the theories of logistics. We will content ourselves with taking, as an example, the definition of irrational number set forth in

chapter XXXIV of Russell's *Principles of Mathematics*. We will sum up what Couturat says of Russell's results: "His definition consists in identifying the irrational number with the *lower class* which previously served to define it.... We will give the name of *segment* to every not null class of rational numbers, which does not comprise all rational numbers, which comprises all rational numbers smaller than any one of its elements, and such that each one of its elements is smaller than another of its elements."[27] These last two conditions may be expressed in symbols. It is shown that there are more segments than rational numbers. "These segments, by definition, will be the real numbers."[28] The irrational numbers has been identified with the lower class.[29] Does this laboriously obtained definition, after criticism of the many definitions of the irrational number, throw the faintest light on the strictly mathematical difficulties which come under the tneory of these numbers? By no means, as we shall at once show.

How far have the preceding definition and the different logistical theories dealing with the irrational numbers helped to solve the following problem?

A number being defined by an infinite succession of integers, to find whether this number is commensurable or incommensurable.[30] True, it is known that if a number is defined by its expansion in decimals, for the number to be commensurable it is necessary and sufficient that the series should be periodic from a certain number onwards. It is also known that if the number is developed in a continued fraction, in order that it may be commensurable the development must be limited. These rules, however, do not solve the general problem.[31] This is not all, the logistical theory does not permit us to recognize whether a definite incommensurable number is algebraic or transcendental, and in case it is algebraic to determine its degree.

As regards more especially transcendental numbers, their existence is by no means evident *a priori*, it results, as it known, from

[27] Couturat, *Les Principes des Mathématiques*, p. 85.

[28] *Ibid.*, p. 86.

[29] We might also identify the irrational number with the higher class.

[30] We shall use without distinction the terms "incommensurable" and "irrational," though both alike are defective. As a rule, the term "incommensurable" is used for dimensions.

[31] E. Cahen, *Eléments de la théorie des nombres*, p. 172 and p. 183.

a theorem of Liouville: p/q being an irreducible fraction, value approaching ξ, if we have

$$\text{Mod. } (p/q-\xi)<1/q^{n+1}, \text{ since}$$

the values of q transcend all limit, ζ cannot be an algebraic number of the degree n. If we can prove this for each value of n, ξ is transcendental.[32] Thus the existence of ζ demands a particular demonstration. We may add that, as regards algebraical numbers, there is a theorem of Lagrange which enables us to know when they are of the second degree. Any reader desirous of studying these questions more deeply will find important results in Edmond Maillet's work on the theory of transcendental numbers.[33] Especially do we draw attention to the theorem which supplies the necessary and sufficient condition for an irrational, positive, real number to be a transcendental number of Liouville.[34] In these investigations no mention is ever made of the formula of logistics. In a word—and it is all we wish to remember of the foregoing explanations—the logistical definition of the irrational and the considerations that logisticians may have advanced regarding it, whether put in formula or not, throw no light whatsoever on the determination of the distinctive characters of the commensurable and incommensurable numbers, of the algebraic incommensurables of the different degrees and finally of the transcendental numbers. In these matters, the results have been obtained by attacking the difficulties directly by the ordinary mathematical methods. Along these lines, we may add that there is much to be done.

It would be fastidious to mention all the main problems not solved by the ordinary methods, and for which logistics gives no help at all to the mathematician. As an example, we will cite the following case found in analysis. "It is known that two functions, which always have the same derivative, differ only by a constant, when this derivative is *finite;* as regards the general case, nothing is known"[35] or, along a totally different order of ideas, we will recall one of Fermat's enunciations, unsolved in the general case: the equation $x^n+y^n=z^n$ is not resolvable into integers for $n<2$. We must not

[32] Borel, Leçons sur la théorie des fonctions, p. 26.

[33] Maillet, *Introduction á la théorie des nombres transcendants,* 1906.

[34] *Ibid.,* p. 124.

[35] Lebesgue, *Leçons sur l'intégration et la recherche des fonctions primitives,* p. 75.

[36] Couturat, *Les principes des mathématiques,* p. 79.

forget, however, that we are writing in a philosophical review, and so we would rather examine a question of a more general character: the generalization of number will enable us to make a few interesting remarks. "The same logical method," writes Couturat, "enables us to explain the generalization of number. First, the rational numbers will be considered as relations between integers. . . . The positive and negative numbers also will be conceived of as logical theory of irrational numbers requires the previous study of the continuous, just as the logical theory of complex numbers presupposes the theory of space.[37] Logistics finds no serious difficulty, so long as it does not more than justify and organze the numbers, or, more generally, the mathematical entities which positive science has adopted. When we are dealing with new entities, however, the scientific use of which is still disputed, it does not appear as though logistics could afford any very effective help in the solution of the questions, so true is it that the introduction of the new entity is mainly justified by the mathematical use to which it is put. First and foremost, logic has no general process of generation for all possible entities—moreover, it seems as though experiment (for instance, the measurement of certain physical dimensions) plays an important part, at least as regards the origin of the new entities. Consequently, whenever a new entity obtrudes upon the attention of scientists, the question that is first asked is whether it is reducible to the known entities, and only special study in mathematics can afford us profitable information on this point. If logistics possessed a restrictive general principle (or a small number of such principles) which the new entities must verify, it would have a simple logical process which would permit of some of them being eliminated. It was thought that Hankel's principle of the permanence of the rules of calculation had this property; but, by a strange irony. it was the logisticians who pointed out that this was not so. This principle, remember, may be formulated as follows: "If, for the purposes of generalization, we can give up some particular property of an operation, we must on the other hand take care not to add any new property to those already used in the restricted operation, so that any rule set up for the generalized operation may also be applicable to the restricted operation."[38] To say that this principle is false means, in a word, that it is not possible to fix for the arithmetical operations general characters valid for all classes of numbers, and more generally for all classes of

[37] *Ibid.*, p. 81.

[38] Houël, *Cours de calcul infinitésimal*, I, 5.

mathematical entities, since each particular class of entities has its own distinctive operative laws. But then, such reasoning as that of Weierstrass, for showing that any hyper-complex number of an ensemble with n units "is decomposable into r complex numbers belonging respectively to r partial ensembles with *one* or *two* units.... the ensembles with one unit being analogous to the ensemble of the real numbers....the ensembles with two units being analogous to the ensemble of the ordinary complex numbers,[39] a reasoning that is possible only on condition that we suppose the associativeness, the commutativeness, and the distributiveness of the multiplication of the new complexes, is still quite formally correct, though it loses all general philosophic bearing since Hankel's principle is false. The field, then, remains absolutely open to the creation of new entities which will individually have to be subjected to a special mathematical investigation. We may add that the relations of the entities with their operative laws have nothing fixed: certain numbers, such as integers, are reproduced by addition and multiplication but others (algebraic numbers of the second degree, for instance), generate numbers of a denomination different from themselves by addition and multiplication.

Of all the new theories, however, the theory of transfinite numbers is by far the most instructive to examine; for the reason that it contains a class of new entities on the bearing of which mathematicians are not agreed. What are the transfinite numbers which must be admitted? Those of the first two classes? Those of a higher class? Are not the *ordinal* types called to a greater future than the *transfinite cardinals?* Must we reject all these numbers *en bloc*? Mathematicians are not agreed, and logisticians, hitherto, have come to no positive result. Still, if we trust to what the history of mathematics teaches us, there is every ground for believing that geometricians, of themselves, will solve the difficult problems contained in the new doctrines. Indeed, the theory of imaginaries and hyper-complexes had given rise to logical or philosophical difficulties over which mathematical thought, without the help of foreign allies, has succeeded in triumphing. In all probability it will be the same with the transfinite numbers. Here we do not claim to go to the root of this problem; we simply state at the outset that the criterion of mathematical utility constitutes, provisionally at least, a principle of elimination which will probably cause many parts of Cantor's work

[39] Couturat, *De l'Infini mathématique*, note I, p. 587.

to be condemned. This criterion has been formulated by Picard in the following terms: "There will be occasion to develop it (the arithmetic of the transfinite numbers) only if these views prove to be fruitful in analysis; consideration of the transfinite numbers has already enabled us to discover certain theorems, though we must say that they might have been obtained by another method."[40] Nor must we forget to make a distinction—one which might escape the notice of philosophers—between the theory of *transfinite cardinals* and that of *transfinite ordinals*. As regards the first theory, the opinion of Borel would seem provisionally to sum up the question best: "The second principle of formation cannot make us acquire the notion of a power we had not already, and it appears doubtful whether we have any definite idea of what a power beyond the second can be.... It cannot, however, be denied that at the present time the expression *transfinitely* has still a less precise meaning than the expression *indefinitely*, the result being that our precise knowledge of the different powers goes scarcely beyond the following remark: there are numerable ensembles and non-numerable ensembles, the later notion being mainly negative."[41] The theory of the ordinal types seems called upon to play a more important part than that of the transfinite cardinals. In this connection, let us consider what led Baire to introduce the notion of transfinite ordinal: "In the same way, for well ordered ensembles, we think it may be advantageous, when fixing the notion of *rank* in such an ensemble, to attach a new term to this notion and proceed as though all the elements of a well ordered ensemble had ranks determined once for all. Now, we have seen that integers were inadequate for this purpose, we shall give them new signs which will be the transfinite numbers."[42] To sum up, it would seem that the determination of the definite balance of the theory of transfinites comes within the province of the mathematicians, and that certain results only of Cantor's theory, but not all, as Russell seems to think, may be retained. Moreover, the existence of logistics, from the above explanations, is in no way connected with the success of the whole of transfinitism.

[40] Picard, *La science moderne et son état actuel*, p. 56.

[41] Borel, *Leçons sur la théorie des fonctions*, p. 122.

[42] Baire, *Leçons sur les fonctions discontinues*, p. 41.

III. IRREDUCIBLE INTEGRALS AND THE NUMBER OF PRIMITIVE IDEAS.

We know that, in problems of quadrature, we are led to consider functions that cannot be integrated by means of algebraical expressions and elementary transcendentals (logarithm, exponential, circular functions) in finite number. Thus, we establish the existence of types irreducible to simpler forms. Then we say that there are new transcendentals. We cannot insist too strongly on the way in which mathematicians study these new entities.[43] The process we are about to characterize in a general way is of considerable philosophical importance, since the so-called chain of purely analytical deductions from mathematical propositions would seem at a given moment to be broken; supposing we are to imagine mathematical propositions as forming a chain. At a certain moment, we find mathematical entities which cannot be formally integrated by means of elementary forms in finite number. We do not on this account banish them from the field of analysis, but we study them as new objects with properties *sui generis*. They will have formulas of addition, multiplication, developments in series, etc., which are proper to them.

Take, for instance, the elliptic integrals which play a large part in analysis, and the question of the reducibility or irreducibility of differential equations....

These irreducible mathematical entities constituting a non-enumerable infinity, how are we to reconcile this fact with the logical theory according to which there is only a finite number of elementary ideas? Are we to say that these original mathematical entities, transcendental numbers, irreducible integrals, form a *progression*? The term progression, however, in the logistical theory, signifies a sequence similar to the sequence of the natural numbers. A finite number of indefinables and of indemonstrable principles suffices to constitute the unlimited sequence of the natural numbers and their elementary properties. (Of course, the logistical doctrine throws no light on the inner nature of these numbers, as we may see by studying the theory of prime numbers.) As regards the natural numbers, the elementary operative rules are permanent for the entire sequence. However large a natural number, even if it extends to a hundred digits, the laws of addition, substraction, etc., will remain the same

[43] To justify our conception there might be found other instances than the one we are examining. This, however, has the advantage of emphasizing the fundamental notion of irreducibility.

for it as for the prime numbers, unless we contest the value of the principle of complete induction. The ensemble, however, of the irreducible transcendentals cannot be regarded as forming a progression. First, the operative rules are no longer retained; for instance, there is a formula of addition for the elliptic functions which is not the same (although these formulas have analogies) as the formula of addition of the trigonometrical functions. Again, there is no *consecutiveness* between the elements of such an ensemble, such as there is in the sequence of the natural numbers. Finally, the power of this ensemble is not like that of the sequence of the natural numbers, the power of the ensemble of the irreducible mathematical entities constituting a non-numerable infinity. On the other hand, however, we cannot think with an infinity of logical principles. No doubt such notions as those represented in mathematics by the signs of an infinite sum, an infinite product, an integral, imply an infinity of elements, but as regards these notions we must agree with Russell and Couturat that they are thought in comprehension, not in extension. It must then be concluded that we think with a finite number of logical constants (indefinable notions, indemonstrable propositions) which enable us to study the transfinitely infinite universe of mathematical entities. We will sum up what we think regarding the number of the elementary ideas by saying that the system of our logical ideas constitutes a closed system, and that the universe of abstract and concrete entities to which it applies will always be for us an open system. But, for that very reason, the study of the universe of entities will never be completed, and it will force us to modify the closed system of the logical constants. We shall indeed be led to consider new irreducible entities, which will not necessarily come within our logical schemes; hence the need to perfect them. At a given moment of evolution, we always think with a finite system of logical constants; but one system succeeds another. Couturat has claimed the right, quite a legitimate one, to transform and improve academical logic, and has successfully refuted the extraordinary theory according to which logic alone, of all productions of the human mind, having its origin in the brain of Aristotle, constitutes an eternal and consequently divine monument. Just because logic, however, is subject to the general law of evolution, we cannot affirm that the logic of Russell, with its nine indefinables and its twenty undemonstrable propositions, possesses that character of perenniality which we rightly refuse to attribute to the work of Aristotle.

UMI

Observe that evolution may take place by way of adjunction—as is the case with mathematics. It is not necessary, as is erroneously imagined, that it should take place by way of contradiction (Hegelian theory). The multiplication table, the formulas of trigonometry, said to have been first formulated by Hipparchus and, dating back several thousands of years, have never been contradicted, never ceased to be true.

IV. THE NOTION OF FUNCTION AND THE RESTRICTIVE CONDITIONS FOR ITS USE IN MATHEMATICS.

In the last two paragraphs we have indicated mathematical theories which are opposed to an illegitimate extension of logistics outside of its field of application. Inversely, we must not restrict this field too much. It would appear as though Pierre Boutroux has gone too far along this line and refuses to logistics the exercise of a legitimate right. In the determination of the conditions that restrict the notion of function, he thinks he has found an extra-logical operation,[44] consequently, a limitation to the domain of logistics Later on we will examine more thoroughly this complex problem which raises numerous difficulties. In this short paragraph we will simply submit to the philosophers certain general results, which will enable us to make a few reservations as to the judgment of Pierre Boutroux.

It may be said that the most general notion of function is a discontinuous and nonuniform function. It is known, however, that analysis does not possess a theory of functions corresponding to this absolutely general type. Indeed, it is easy to show that we cannot at present set up a theory of the most general discontinuous functions. Let us follow Borel's reasoning: a continuous function may be given by means of a numerable infinity of conditions; it is not so with a discontinuous function in its most general meaning: such a function is defined by a *non-numerable* infinity of conditions; in practice, this is equivalent to saying that it is impossible to define it."[45] Consequently, we cannot think of approaching the theory of functions by allowing this notion to retain its greatest generality. Restrictions must be imposed on it.

Keeping to the notion of function, Lebesgue has retraced the history of the early transformations of this notion. We will briefly sum up his statement. At the time of Newton and Leibnitz, a

[44] P. Boutroux, *Revue de Métaphysique et de Morale*, July 1905, p. 633.
[45] Borel, *Leçons sur la théorie des fonctions*, p. 126.

function was generally called "a quantity y connected with a variable x by an equation, with a certain number of symbols of operations intervening (arithmetrical, trigonometrical, logarithmic operations.[46] Then, following on the problems of integration, we have been led to consider the case in which there is a geometrical relation between a function S (x) (*area*) and x. Afterwards, a distinction was made between "the geometrical figures defined with the aid of laws capable of being expressed by geometrical equalities and the figures that were not thus defined."[47] To the curves included in the first case corresponded the continuous functions (Eulerian continuity), to the curves of the second kind corresponded arbitrary functions which were not true functions." The continuous functions were the true functions."[48] Later on, Fourier overthrew this method of considering the functions, by showing that the "trigonometrical series which could be employed, in many cases, for the representation of continuous functions, might also be used for the representation of non-continuous functions formed of parts of functions."[49] Then Cauchy gives the following definition of function: "y is function of x when to each state of magnitude of x there corresponds a perfectly determined state of magnitude of y."[50] To Cauchy's mind, it was necessary that these correspondences should be analytical; later on, this restriction was removed. Without dwelling upon the important investigations of Riemann and the subsequent studies which have thrown light on this matter, we may mention summarily the results obtained by Baire. Baire asked himself what were the conditions which the discontinuous functions must satisfy to be representable by series of continuous functions. He first shows "that a function presenting *a finite number* of discontinuities is the limit of continuous functions."[51] In a more general way, he distinguishes the *punctually and totally discontinuous* functions, according as their oscillation has its minimum everywhere nil, or not.[52] (We know that the oscillation is the difference between the maximum and minimum values of the function in an interval.) He obtains the following theorem: "Every

[46] Lebesgue, *Leçons sur l'intégration et la recherche des fonctions primitives*, p. 1.

[47] *Ibid.*, p. 3.

[48] *Ibid.*, p. 3.

[49] Lebesgue, *Leçons sur l'intégration et la recherche des fonctions primitives*, p. 3.

[50] *Ibid.*, p. 4.

[51] Baire, *Leçons sur les fonctions discontinues*, p. 11.

[52] *Ibid.*, p. 75.

limit of continuous functions, is a punctually discontinuous function."[53]

After this too brief statement, we may say that the problem set by Boutroux is intimately connected with the history of the notion of function. Now, this history has never been specially and fully set forth. Consequently, so long as the equivalent of the historical work of Charles on the methods of geometry does not exist for the theory of functions, we cannot clearly deduce the part of real logic implied in the fundamental notion of function and in the conditions imposed on it. When Pierre Boutroux affirms that "the conditions by means of which we determine the idea of function have an indeterminate character, that they are not the known but the unknown quantity of a problem,"[54] or even that investigation into the conditions to be imposed on the functions constitutes an extra-logical operation, he adopts the point of view of discovery. First, however, the notion of function is with difficulty distinguished from that of *relation*, which seems to form part of those grammatico-logical notions mentioned at the outset of this study as forming part of the domain of logistics. Do not the most general conditions, at least, which we impose on the idea of function, also share to some extent in this logical character? We have just indicated the method which, by unfolding the historical origin and the developments of each notion, would in our opinion permit of a reply to this question; we cannot here develop it further.

V. CONCLUSION.

We will briefly sum up the conclusions drawn from this study.

Note first that the controversy, to which we alluded at the start, ceased to deal with the essential philosophical problems when it degenerated into an examination of the cases of mathematical *teratology*. The Burali-Forti antinomy, the Richard contradiction, the "All Cretans are liars" sophism are the extreme confines of the science. We do not deny that they are interesting, but there are more fundamental problems lying at the very heart of mathematical theories. To resolve to reduce the philosophy of mathematics to the solution of sophisms would be as unjust as to look upon pathological physiology as the model of all physiology. Now, the readers might have this impression on reading the latest discussions dealing with

[53] *Ibid.*, p. 83.

[54] P. Boutroux, *Revue de Métaphysique et de Morale*, July 1905, p. 628.

these questions. We may even go further and say that the elaboration of logistics does not involve all the difficulties which deserve to create reflection on the part of the critic of the sciences, and that there are real mathematical problems which give rise to philosophical questions of wide range.

Afterwards, we saw that logistics constitutes a restricted though indispensable branch of general mathematics.[55] Remember that Auguste Comte, who is usually quoted as an opponent of formal logic, conceived the possibility of this about the year 1830: "I do not know whether subsequently it will become possible to give *a priori* a genuine course of method altogther independent of the philosophical study of the sciences, but I am convinced that this cannot be done at present, the great logical processes not yet being capable of explanation with due precision, separately from their applications. I also make bold to add that, even if such an attempt could subsequently be realized, which indeed may be regarded as a possibility, this would still only be effected by the study of the regular applications of the scientific processes...."[56]

The legitimacy of logistics, in its own sphere, can no longer be seriously denied; the only thing remaining is for us to find out how far, *in reality*, it has fulfilled its task. We may repeat, however, that it cannot claim the rôle of a new method of investigation; to compare logistics with the differential and integral calculus, for instance, would be altogether erroneous. From its beginning, the differential and integral calculus has afforded a solution of mathematical problems: problems of the quadrature and the tangent, more special problems such as that of the catenary. There is nothing of this in logistics, which remains the mathematics of the elements. Nor could it be regarded as a sort of universal speciousness, wherein mathematical thought would be wholly done away with.

To sum up, there are two conceptions of logistics: the one which assigns to it a special positive function, the other which, desiring to take up in it the whole of human thought, transforms it into an algebraical scholasticism as barren as that of the Middle Ages. As

[55] Peirce maintained that logic is a branch of mathematics, M. Couturat claims that mathematics is a branch of logic (a branch far more important than the trunk from which it sprang). The question thus formally stated appears to be of no great practical or scientific interest. The main thing is that there is a branch of positive studies called logistics, that this is studied in the form of a symbolical calculation, a calculation whose efficacy is *special* since it cannot solve problems in other branches of mathematics.

[56] Auguste Comte, *Cours de philosophie positive*, 1830 edition, I, 39.

we have seen, scientific thought comes up against real problems, facts in the solution or explanation of which the reduction of the ordinary mathematical methods to the logistical methods afforded no advantage whatsoever. Logistics, *qua* universal explanation, should be condemned by the same right as any kind of metaphysics, because, being practised outside of its field of application, it remains a mere sport of the mind, of no scientific use whatsoever. Indeed it is one thing to determine and to classify the grammatico-logical elements of ordinary language, which form an element in mathematics; it is another thing to claim absolutely to reduce all mathematics to these elementary forms. In the former case you act as a scientist, in the latter as a metaphysician. The complete identification of mathematical thought with the grammatico-logical elements which condition it is an illusion analogous to that of materialistic dogmatism, which wholly assimilates thought to the elements of the brain, its material conditions of production. The proof that this identification is ineffectual results from the fact that, if logistics made the mechanism of mathematical thought absolutely transparent, and if it practically absorbed this thought, we should find no more mathematical problems offering difficulties, which logistics would not be capable of solving immediately. Now, we know that nothing of the kind takes place.

In reality, we are confronted with an ensemble or irreducible methods and data which form the various branches of mathematics. In no way could we explain all this knowledge by a systematic philosophical view. At all events, such an explanation would have no objective value scientifically. We may, however, bear witness that scientific methods are organized and transformed according to the ideal of the logician, and that, in theoretical sciences, mathematical demonstration surpasses all other forms of thought (intuition, imagination, etc.). By studying the inner mechanism of this demonstration, we glimpse the logical meaning of "the mysterious unity manifested in analytical works apparently most remote,"[57] a unity the complete realization of which could not be regarded as effectively brought about, but only as ideally conceived, at the present stage of our knowledge.

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[57] Hermite, *Comptes rendus de l'Académie des Sciences*, 1862, Vol. LV, p. 91.

MAN AS A MUTANT.

The failure of theology to give a comprehensive meaning to human existence was the cause of speculative philosophy. System after system, from Thales down to Bacon, attempted to explain the phenomena of nature and their relation to humanity. With the advent of the scientific era, philosophy was compelled to limit its speculations so as not to contradict the results of science and philosophy. Empiricism in philosophy became of great importance, and science became philosophic. Spencer defined philosophy as the generalization of the sciences. Science, in its turn compelled humanity to change the ideas of creation by proving that matter cannot be created or destroyed, and substituted evolution for special creation. These new ideas, coupled with the overthrow of the geocentric theory of the universe, revolutionized the whole of human thought, as shown by modern philosophy, art and literature.

Most of the philosophical systems, especially the idealistic ones, are optimistic in their nature. They all hold that nature, or its god, is conducting the universe with a special purpose in view, and that man, as the most important part of nature, has a special destiny to perform which is for his good. There are a few exceptions. Kant, although an idealist, does not show much optimism. He, like Spencer, sees constant change in nature, but unlike Spencer, he is not so sure that the change means progress. Schopenhauer developed a doctrine which may be described as a transitional form from the idealism of Kant to the prevalent realism of the present. His pessimism is derived from human experience. He emphasized with unusual lucidity the feebleness of the spontaneity of the intellect. He comes to the conclusion that not only is our world not the best, but it is the worst of all possible worlds.

The generalizations of science have a distinct bearing on the questions raised by theology and philosophy. Science in its prevaling form teaches that there can be no such thing as design in nature. Man, together with all organic life, must adapt himself to environment in order to continue his existence; and the end of all man's strivings on earth is doomed to failure. In a word, the message that science brings is woefully pessimistic. It tells us: Try as you may, make new discoveries, new inventions, dig as deep as you can in the

secrets of nature, you are one and all predestined to destruction. The following are some of the facts adduced by science in support of this gloomy foreboding.

It is well known that organic life is made possible by the energy radiated on this globe by the sun. There can be no life (except of the lowest forms) without solar energy. Science tells us that the sun's energy is diminishing continuously. Ergo, the time must arrive when the whole of the sun's energy must be exhausted. Long before that time arrives human beings will have ceased to exist. The range of temperature in which protoplasm is able to exist is very limited. The range of temperature favorable for the existence of higher forms of life is still more limited. No human being could possibly hold his own at a temperature of 60' centigrade below zero, while the absolute temperature to which the sun's energy can descend is 273' below zero. This shows that animal life on this planet must cease long before the sun is entirely extinct. The simplest experiment to show that the sun's energy is constantly diminishing is shown by an instrument known as the pyrheliometer. It is a small, hollow cylinder, presenting at one end a disk of known dimensions covered with lampblack. The body of the instrument is made of silver, well polished to prevent radiation of heat, which it absorbs through the blackened disk. The disk is turned directly to the sun. The cylinder is filled with water. All the radiant heat and light, everything in the form of radiation that falls upon the lampblack, is absorbed by it and is degraded to the form of heat energy and so communicated to the water, which contains the bulb of a thermometer, whose stem runs through the axis of the apparatus. By a simple contrivance the apparatus can be so adjusted that the blackened disk, which is always capped when not in use, will receive the sun's rays perpendicularly. The cap is removed for a measured period of time and then put on again. The instrument is shaken so that the temperature of the water remains the same throughout. The rise in temperature is shown on the thermometer. Correction is made for the loss of heat during the performance of the experiment. In this way a fairly approximate estimate of the amount of heat received from the sun in a given time can be made. By comparing the surface of the disk with the surface of the whole earth which is exposed to the sun, we can estimate how much radiant energy in the form of heat, light, actinism, and so forth, comes to us from the sun per second of time; and we can also estimate the amount of energy that leaves the whole

sun's surface every second. By many years' observation it has been conclusively shown that the average amount of energy reaching the earth is diminishing gradually. It is true that the loss is very small, on the highest estimate not more than 1' centigrade in seven thousand years, but the loss is positive; which forces the conclusion that it is only a matter of time when the earth will no longer be fit as an abode for man.

This, however, is not all. As if to make the escape doubly impossible, there is another, more general phenomenon pointing to the inevitable loss not only of life, but of all activity on earth. Our planet, in conjunction with the whole solar system, is bound to become a dead body, uniform in temperature and devoid of all motion. A thorough discussion of this idea would lead us to the subjects of Entropy and The Theory of the Potential, subjects out of place in an article of this nature, on account of the mathematical treatment that they involve. I shall, therefore, have to limit myself to a mere indication of the nature of the argument. It is based on one of the most fundamental laws of physics known as the law of the conservation of energy. The law states that energy may change its form, as mechanical into heat energy, but no energy can be a constant quantity. Experience teaches us that every change of energy from one form into another is accompanied by a loss of "available" energy. No energy was lost to be sure, but some energy has been degraded, i. e., it came down from a higher to a lower potential or level. In illustration of this principle it would be very desirable to bring in Carnot's Cycle and the subject of reversibility, were it not for the fact that it would divert us too far from the main issue. It will suffice to state that after every physical operation in the universe the whole amount of energy involved cannot be recovered in an available form. A considerable amount is lost in friction, radiation, etc. It is well known that even the most efficient steam engine converts less than one-fourth of the heat which the coal generates into useful work. The remaining three-fourths are lost, not to the universe but to man, and are thereby rendered "unavailable." As there is no process in nature which is not accompanied by a loss of "available" energy, the conclusion is forced upon us that a time must arrive when all the higher forms of energy, i. e., energy of motion, energy due to position, chemical energy, etc., will be degraded into one form of energy—radiant energy. The amount of energy will still remain constant; there will be no loss of energy, but it will all

become unavailable. Energy will reach its lowest level; after which no activity of any kind will be possible. This conclusion is a direct deduction from the second law of thermodynamics. The law is stated by Clark-Maxwell as follows: "It is impossible to transform any part of heat of a body into mechanical work except by allowing heat to pass from that body to another of a lower temperature." But as there will be no body of lower temperature in the universe it follows that all activity must cease. It is true that Clark Maxwell, the greatest mathematical physicist since Newton and Laplace, to whom all the great development in electricity, including wireless telegraphy, are due, thought that he had found a way out of the difficulty. He conceived a being whose faculties are so sharpened that he can follow every molecule in its course. Such a being, known in physics as Maxwell's Demon, could sort the molecules so as to allow the swifter ones to go into one compartment and the slower ones into another. He could then, without expenditure of work, raise the temperature of one part of the universe and lower that of the other, thus starting a difference in potential and causing new activity. I mention it here simply as an ingenious curiosity.

There is no denial of the fact that all of this is exceedingly pessimistic, and yet, as such, it must be accepted. But in all this darkness there is a gleam of light which makes it worth while for humanity not only to continue its existence but to consider its being on earth as a rare and special privilege. It is quite probable that in the whole universe man is the only being who has the privilege of a conscious existence, who is possessed of the power to analyze and synthesize objectively. It is quite probable, nay almost certain, that his conscious existence came accidentally, and that as a conscious being he is the only creature able to understand nature and study her so as to foretell phenomena centuries in advance of their appearance. All students of nature are aware of the fact that the universe is composed principally of inorganic matter. Organic life occupies an insignificant part of the time of a planet's existence. It takes untold centuries before a planet, even when favorably placed in the solar system, is able to support life. After a brief period comparatively speaking, it becomes lifeless like the moon and remains so indefinitely. A glance at our solar system will show that of all the planets that are engaged in their endless cycling around the sun only one—Mars—besides the earth may be said to be favorably placed for life formation; and if Mars possesses life, the inhabitants of Mars must

be inured to somewhat severer conditions than generally prevail on earth. The other planets are either too near the sun, where the temperature is too high, or too far, where it is too cold and dark, or they are in a gasious state so as to preclude the existence of life, especially in any high form. In this connection I will cite a paragraph of J. F. W. Herschel, one of the greatest astronomers that ever lived. Speaking about the probability of animal life on the other planets, Herschel analyzed the following three features: "First, the difference in the respective supplies of light and heat that the planets receive from the sun. Secondly, the difference in the intensity of the gravitating forces which must subsist at their surfaces, or the different ratios which on the several globes, the inertia of bodies must bear to their weights. And thirdly, the difference in the nature of the materials of which, from what we know of the mean density, we have every reason to believe they consist. The intensity of solar radiation is nearly seven times greater on Mercury than on earth and on Uranius 330 times less; the proportion between the two extremes being that of upwards of 2000 to 1. Let any one figure to himself the condition of our globe were the sun to be septupled, to say nothing of the greater ratio! or were diminished to a seventh or to a 300th of its actual power! Again, the intensity of gravity, or its efficacy in counteracting muscular power and repressing animal activity, on Jupiter, is nearly two and a half times that on earth; on Mars not more than one-half, on the moon one-sixth, and on the smaller planets not more than one-twentieth; giving a scale of which the extremes are in proportion of sixty to one. Lastly the density of Saturn hardly exceeds one-eighth of the mean density of the earth, so it must consist of material not much heavier than cork. These physical peculiarities show how improbable it is for higher forms of life to exist on the other planets of our solar system. It may, however, be argued that there are many more systems scattered through infinite space where the planets are as favorably placed to their illuminaries as the earth is to our sun. Granting the universal law of evolution, we must admit that those planets must also reach a state when their physical conditions are favorable for the support of life. Hence they too must go through the same cycle of evolution as our earth, reaching as high a state of mental development and even higher. In fact this is now the accepted belief of most scientists Proctor, reasoning on these premises, comes to the conclusion that in any given time it is safe to assume that no other life exists on

other planets than the earth. He bases his conclusion on the fact of the insignificant duration of life on any planet as compared to its lifeless existence. Given any planet, the probabilities are that either it has not as yet reached the state of development capable to sustain life—as Jupiter or Saturn—or the life sustaining period is all over, as in the moon. This is sound reasoning. It is based on the law of the Uniformity of Nature and on the Theory of Probability. This tacitly assumes the existence of life, in all its forms, on every planet during a certain period of its development.

Reasoning, however, from different data, that came to light since Proctor's essay was written, it seems to me that there are strong grounds that preclude such an assumption. According to Darwin, the animal kingdom has developed very slowly from the lowest organism into the highest forms of life. The time required for such development was recognized by Darwin himself to be very enormous. This great length of time would not matter in an ocean of infinity, had there been no objection from other quarters. Both astronomy and geology in calculating from their respective data the probable age of the earth, fall short of the time necessary for the development of the different species on a planet ripe to support life. It is also interesting to note that, although starting from different considerations, the conclusions reached by both astronomy and geology as to the probable age of the earth is almost the same. There is good reason to believe that Darwin forsaw this difficutly but could not account for it. Lately, however, new light was thrown on this subject by the investigations of Hugo De Vries. In his "Experiments and Observations on the Origin of Species," De Vries propounds what is known as the mutation theory. According to this theory life has not developed gradually and slowly but it came in leaps and bounds. Occasionally some new species appears, sometimes not at all resembling the parents and very often of a much finer and higher type. This new species continues to propagate its own type without ever returning to the lower characteristics of the ancestors. By accepting this theory (and it is now universally accepted by biologists) the time for the development of life on earth is very much reduced, so as to fit in with the calculations of the astronomers and geologists. That man is a descendant of the simian family is accepted by almost all biologists. New evidence has lately been added from quite different sources which show conclusively the relationship of the human species to the anthropoid apes. From

a morphological standpoint it is admitted that there is a missing link between the highest ape and man. If this is true from the morphological standpoint, it seems to me that from a psychological viewpoint not one but a thousand links are missing between the higher animals and man. I am perfectly aware that animals possess some intelligence, and even foresight, but they do not possess to any considerable degree the objectivity, the contemplation, the conscious thought, in a word, personality, that are peculiar to the human. Not only do they not possess these qualities, but there is no evidence of their ever being able to acquire them. Huxley made the statement that his dog was more intelligent than the Australian Bushman because the dog could count up to four while the Bushman could not count more than two. If a group of more than two be presented to him he calls it "many," no matter how many units it contains. But we must take into consideration the potentialities of the Bushman. As far as I know nobody has ever succeeded in teaching a dog reading or writing, nor an anthropoid ape arithmetic or algebra. Most people would agree that such a task would prove impossible. On the other hand, there are good reasons to believe that the Bushman, through many generations of training and education, could be brought up to a high degree of culture. Circumstances have kept him in a savage state; but apparently he possesses the potentialities of the civilized human. He possesses a remarkable faculty for graphic illustration. The Bushmen show some skill in their drawings of men, women, children and animals. They are valued as servants by the Boers for their possession of sufficient intelligence to perform their work properly.

This almost infinite gap in intelligence between higher animals and man can only be explained by the mutation theory. In analyzing the physical features of anthropoid apes and man Metchnikoff infers "that man is a case of the arrested development of some simian of ancient days, as it were, a simian monster from the zoological point of view, altho not from the esthetic. Man may be regarded as a prodigy sprung from an ape, born with a larger brain and an intelligence more highly developed than occured in his parents." His appearance on earth, then, was sudden, as an accident. As such an accident he is the sole exception to all life on the globe. He is the only self-conscious animal, planning for his future, prophesying events in nature a hundred years to come, able to analyze the composition of worlds almost an infinite distance away, trying constantly

to work out a philosophy of life and rules for his individual conduct and his relation to his fellow men. He is not only the sole self-conscious animal on earth, but the probability is that he is the only one in the whole universe. Before the mutation theory was known it was quite logical for Mr. Proctor to conclude that in the course of evolution every planet favorably situated goes through a development similar to our earth, and that a time arrives, though of comparatively brief duration, when life, like the one on earth including man, must develop. With the advent of the mutation theory this conclusion becomes invalid. An unforeseen species may spring up unexpectedly, as there is good reason to believe in the case of man.

It was Simon Newcomb who suggested that the stellar system was not infinite, but finite. He based his theory on the observation of the receding of the clusters of stars in the Milky Way. Taking our sun with its attendant planets as the center, we observe that the stars in the Galaxy recede symetrically in all directions until a region is reached where there are hardly any stars visible. As the decrease of the stars from the center to the peripheri is in a convergent series, a place must be reached where there are no stars at all (as any convergent series has zero as its limit.)

Newcomb took up the other alternative, that there might be other galaxies which are so distant, that they are invisible to our telescopes; so that instead of one stellar system there may be a great number, even an infinity of stellar systems. This idea was first suggested by Kant, who believed that the most distant nebulae constituted other Milky Ways or stellar systems as extensive as ours. Although he admits such a possibility but from the symmetry of the arrangement of the stars and other considerations, he shows that the probabilities are against such a supposition. Consequently our stellar system is the only one, and is therefore finite.[1]

Riemann's Geometry, which postulates a finite universe, may lend color to the argument. Although Riemann was a pure mathematician, there are many examples in the history of science showing that the visions of theorists have proved true prophecies of nature. or, in the words of Schiller

> "Mit den Genius steht die Natur in engen Bunde
> Was der eine verspricht, leisted die anderegewiss."

[1] "Theorethical researches of Einstein and Weyl make it probable that space which remains beyond (the visible universe) is not illimitable; not merely the material universe, but space in itself is perhaps finite." A. S. Edington, *Science*, Sept. 10, 1920.

Now, the finity of the universe being granted, the chances in a finite universe for another intelligent human, as the one on earth, to spring up by accident, where an infinite number of excluding combinations of probabilities are involved, is in the ratio of one to infinity, which equals zero.

[It may be argued that if an accident happened on earth why could not another similar accident happen on another planet? Why is it not probable that by chance in other planets high forms of life spring up, even higher than man?

If man were a natural development of evolution we would expect the same result on every planet in the course of its development. Once, however, we regard him as a creature of accident, the chances from a mathematical consideration of his coming on other planets in a finite universe, has been shown above to be equal to zero.] If man, then, happens to be the sole creature in the universe of organic life, and inorganic matter, who is capable of conscious objective thought, there is no reason for an utter pessimism. The very fact that he is seeking for his place in nature and conscious of his surroundings is sufficient reason for his being happy. He knows that he is alive and thinking. Though the days of his life be few and troublesome, he is still the only seer who came accidentally to this privilege and is conscious of his destiny. He is the only one able to plan and work for a brighter life for those of his kind who will come after him. This thought ought to keep him happy and inspire him with courage even during misfortune and disease. It is still better to suffer and know than to become non-existent and be absorbed into the infinity of inanimate matter. In the words of Poincare: "Geologic history shows us that life is only a short episode, conscious thought has lasted and will last only a moment. Thought is only a gleam in the midst of a long night. But it is this gleam which is everything."

His objective attitude toward nature enables him to protect himself against noxious elements in his environment, and by the mastery of her laws to succeed in adapting her to his needs and pleasures, not to mention the greatest of all satisfactions that he derives from the very act of plying into nature's secrets and the admiration of her as an aesthetic phenomenon.

A. A. HIMWICH.

ARISTOTLE AND THE CRITERION OF TRUTH.

In these days of subjectivist theories of truth, when every philosopher's attitude on truth is canvassed, it is interesting to recall the attitude of Aristotle. He, too, has been assigned to the subjectivist position, with what correctness we shall see. Not that Aristotle dealt with the question of the criterion of truth as we now discuss it, but that noetic problems had to be implicitly considered by him as by every systematic pre-Kantian thinker, and that we may gather, from his teachings in various places, what were his views on truth and certainty in general. These last may not always be so explicit as might be wished, but they may be sufficient to show that his criterion of the truth was, objective evidence. Aristotle held that reality had a permanent or essential nature, and that this can be grasped by thought. On the accord of our judgment with the objective order of things, the truth of our knowledge in his view, depends. He takes an objective view when he tells us (*Met.* VIII c. 10)[1] that that which is, in the strictest sense, entity, is what is true or false, and that, in the case of things, this consists in composition or division. This latter insistence he repeats in the *De. Int.* c. 1. He maintains that the alternative, true or false, constantly presupposes a combination or *συμπλοκή*, for of words spoken without connection, none is either true or false (*Cat.* c. 8.)

But there are more important things for our purpose to be said. Aristotle tells us there are first truths which carry our assent (*πίστιν*), not by means of other truths, but of themselves. It is not necessary, he continues, in the case of the principles of knowledge, to ask that by which, but each of the principles has the proof (*πιστήν*) of being in itself (*Top.* I. c. 1).

Aristotle seems plainly to point to the principle of certainty as something objective, the first truths being regarded as determining our assent. The concern of Aristotle with the *isness* of these first truths or principles is primarily essential, rather than evidential. It is strange that Sir Wm. Hamilton should have taken this passage as ground of his affirmation that "Aristotle founds knowledge on belief, and the objective certainty of science on the subjective necessity of believing." The error appears to have arisen from the oc-

[1] Sometimes called Book IX.

currence of the word πίστις, which seems clearly not meant in the frequent sense of faith or belief, but, as in occasional usage, in the sense of a reason or proof, a means of persuasion or assent. There is a similar use of πίστις by Plato for proof or argument, in *Phaedo* (70 B), but I leave out of account the usage of Greek theological writers.

It is a great injustice to Aristotle when Hamilton erroneously says "he founds knowledge on belief," since knowledge never rests on mere belief, however belief may accompany or even outrun it. Aristotle founded knowledge on fact as known in perception, and on principles grounded in reason. Not on some vague thing called "belief" did Aristotle found knowledge, but on first principles which could not be denied, which necessarily conditioned all other knowledge, and which did not presuppose other principles higher than themselves. He based knowledge, further, on dialectical induction, and logical demonstration, as he plainly intimates (*Anal. Post.* II. c. 25). He presupposed experience (*Anal. Post.* I. c. 14; *Met.* I. c. 1), and reached out after knowledge proper—knowledge that was to him scientific—in the comprehension of the universal by means of thought. Such scientific knowledge meant objective cognitions, that is to say, it was of the object *qua* object.

In another instance, Hamilton takes πίστις in the sense of faith or credence, where again it seems clearly to signify proof or argument. That is where, when speaking of the diverse opinions as to rest and motion, Aristotle says that "against all these one proof (πίστις) is sufficient, for we see that some things are now in motion, and now at rest" (*Phys. Aus.* VIII. c. 3). It seems beyond doubt that evidence, and not anything subjective, was the principle of certainty to Aristotle. He connects the truth of our concepts with the reality of the object, not with the satisfaction of our nature. Evidence is primarily concerned with the object, and its intelligibility. We may say that Aristotle's objectivism was that of his time, but objectivism it was. Besides, the expression "we see" in the passage might have suggested to Hamilton that πίστις was not being used in the subjective sense which he supposed, since faith is opposed to sight.

Again, Aristotle says the understanding seeks the truth by judging of a thing conformably to its own essence (*De. An.* III. c. 6.)

This again seems an invocation of objective evidence in the search after knowledge of the truth, even though we are here in a

more abstractive region, where some may prefer to take a more purely abstract or intellectual view of the universal. The thing cannot exist apart from its essence. The essence was still the essence of the particular, in which last the universal was immanent, for Aristotle will not have essence and phenomenon torn apart. For the essence, though ground of the particular phenomenon, is not purely an *ens rationis* for Aristotle, since it is real and actual only in the particular phenomena themselves. Even Rosmini insisted that essences are "objects" and "truly exist, at least as objects of the mind." Whether Aristotle's positions, I may add, as to knowledge being of the individual and knowledge being of the general, are so impossible to reconcile as Zeller and Benn suppose, is, I think, open to question.

An important passage for our present inquiry is that in which Aristotle, speaking of the principle of contradiction, says that the most certain principle of all is that concerning which there is no possibility of deception, for such must needs be that which is most known of all (*Met.* III. c. 3; X. c. 5). In this connection *Anal. Post.* II. c. 15, should also be considered, where in certain passages the denial of such principles as that of contradiction is taken to make knowledge impossible. But if the principle which is most known is to be taken as that which is most certain, it would seem that, by Aristotle, the certainty of this principle—that you cannot affirm and deny being at the same time—was, in its self-evidence, taken as criterion of the truth. The possibility of certainty underlay his whole doctrine, however little he may have engaged himself with certainty as a problem. But it is objective certainty he is concerned with, that of the truth in itself of the object or principle, not anything of the purely "subjective" character spoken of by Hamilton. For it is in the thought-truth itself—not in our subjective thinking—that objective evidence is in the first instance grounded; that is just what renders it criterion of the truth and principle of certainty. There is an evidence of the thing which is nothing but the truth itself evident, and it is of that I am speaking; but there is also an evidence, which is in us, and consists of our knowledge of evident truth. Every true thing is evident; but a thing is not evident for us by its simple truth, but because our intelligence can reach the truth. And the truth itself Aristotle did not confound with our criterion of the truth. Certainty neither denotes knowledge, nor is the equivalent of knowledge, where mere subjective

certainty is concerned. But knowledge does include certainty. Truth, however, is not yet certitude. Certitude is the relation of truth to knowledge, and certitude rests on subjective and objective factors. Knowledge is not simple certainty, but certainty based on cognition. Certainty involves persuasion of the truth, but not the truth itself. A position is objectively evident when the understanding judges that it is necessarily so, and cannot be otherwise. It is one to which we cannot refuse assent, unless we disavow our reason. It is in the spirit of Aristotle that Aquinas, in his work on the Sentences, says "—Certitudo, quae est in Scientia et intellectu, est ex ipsa evidentia eorum quae certa esse dicuntur."

For Aristotle, all ultimate grounds of knowledge were intrinsic, necessary, self-evident. It is of interest to note, in respect of the passage in *Met.* III. c. 3, that that skilled interpreter, Alexander of Aphrodisias, says that, for Aristotle, the principle of contradiction was "known by itself, evident, and indubitable." The self-evident certainty of the first principles or *Grundsätze* was a prime concern of Aristotle, for on them he held all science to depend. "It is necessary," he says, "that the first principles of things, always existing, be always most true" (*Met.* I. c. 1; *Nic. Eth.* VI. c. 7, 2-3).

But their truth must be manifest to our reason; this means their evidence, so far as we are concerned. Aristotle objected to the theory of Heraclitus that, in "affirming all things to be and not to be," it "appeared to make all things true" (*Met.* III. c. 7.) And he maintained the need, in all learning and instruction, of antecedent principle or conviction (*Anal. Post.* I. c. 1.) Also that in the *reductio ad absurdum* mode of demonstration, it is assumed that everything must be either affirmed or denied (*Anal. Post.* I. c. 11.) To him, the principle of demonstrative reasoning may, in itself and in its nature, be the better known, but the inductive mode may be the better known to us (*Anal. Prior.* II. c. 25). It is what is known in itself that I am here concerned with. Things, in Aristotle's view, imposed themselves on thought, and knowledge sprang from the agreement between them. Thought and its object were to him one (*Met.* XI. c. 7; *De An.* III. c. 4.) But this does not mean that reason is overborne in the process, for he tells us that reason is such as to be apprehensive and creative of all things (*De An.* III. c. 5). And, objectively, "all order is reason" (*Phys. Aus.* VIII. c. 1). But he admits that the difficulty of reaching truth "may not be so much in things themselves as in us" (*Met.* I. c. 1).

Another passage may in this connection be referred to. Aristotle says that "not every problem, not every thesis, need be examined" (*Top.* I. c. 9.); for, he goes on to say, there are things that are beyond doubt, such as honor of the gods and love of ancestors. The ingenious mind of Alexander of Aphrodisias was not slow to put forward that in this passage Aristotle was finding the principle of certainty to consist in the evidence. It is at least in keeping with what Aristotle elsewhere urged as to the nature of self-evident truths, whatever may be thought of the examples chosen. But Aristotle thinks those who doubt in the cases mentioned deserve castigation, while those who doubt that "snow is white" merely show false sense or wrong perception.

Confirmatory of what has already been advanced is the fact that Sextus Empiricus plainly intimates (*Adv. Math.*) that two criteria of truth were held both by Aristotle and by Theophrastus, viz., perception for sensibles, and intelligence for intelligibles, and that Theophrastus, more explicitly, put forward evidence as the criterion common to both cases. But Sextus also says that the Stoics recognized two criteria of truth, viz., the sensible and the intelligible, the former true only relatively to the latter. But though the Stoics imitated Aristotle, they were less able to handle the problem. They saw the need for certainty, but without discovering the mode of its attainment. They tended too much to make the distinct sensuous impression the criterion of truth. And yet, like Aristotle, they tried to found science on reasoning, and went beyond him in the lessened place they accorded the universal. But Stoic thought vacillated on the criterion to its own disadvantage. But "the criterion of the Sceptical School," says Sextus, "is the phenomenon, and, in calling it so, we mean the idea of it." He says, further, that we consequently live by giving heed to phenomena, observing these in daily life in an unprejudiced manner (*Hyp.* I. c. 11.) Aristotle, no doubt, held the truth of sense-perception, but simple perception does not suffice to give certitude; there must be a judgment, and a judgment is not contained in the perception as such; only the light of reason renders us certain of what sense attests. Aristotle himself taught that truth belongs only to judgments which represent the way in which real things are conjoined or divided. He who so thinks has the truth, in his view. But for the sensible phenomenon he only claims a certain truth for him to whom it appears, but not truth absolutely. Sensuous perception

does not give us certain knowledge—knowledge of the particular only, not of the general (*Anal. Post.* I. c. 31). It is for him ground of the higher knowledge of principles, through the intellect or understanding.

I have not been concerned either to justify or to criticise Aristotle's positions as to the criterion of truth, but merely to state or elucidate them, mainly from his own works. I think Aristotle held, though not perhaps in so many words, that the criterion of truth does, in fact, exist, and is found in that objective evidence which determines our assent, and engenders in us certainty of the truth. For him, the evident was true, self-manifesting in variant forms in the realm of primordial fact. Such, I think, was to him the objectivity of truth—truth grounded in the nature of things, and prior to all formal truths or judgments.

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PHILOSOPHIC POSSIBILITIES IN SPACE.

It has been suggested, supposed and even emphatically stated that the universe is finite. The most recent statement concerning the limited nature of the universe was made by Professor Einstein, the modern mathematical wizard. Einstein has traveled in thought through the abysses of space where he discovered new and startling laws and theories of light and gravitation.

It somehow seems contradictory that a mind which is so wonderfully capable of detaching itself from that which is local nevertheless conceives the universe to be a finite something. Yet there are many such minds. One reads occasionally about an astronomer who expresses the hope that some day a giant telescope may reveal the limits of the universe to the eager gaze of man.

Of course, we do not always make ourselves clear as to the definition of the word, universe, which we have in mind. Did Pascal refer to the immensity which consists of matter *and* space when he observed that the universe is an infinite sphere the center of which is everywhere and the circumference nowhere? Does Einstein consider solely the world of suns, planets, stars and comets when he

states that the universe is finite? Does he leave space entirely out of his calculations?

If Einstein does ignore the existence of space, he presents us with the following extra-ordinary spectacle: a cluster of stars and a milky way that whirl in unutterable solitude in an infinite vastness of space. This mass of sun-systems is as a pin-head lost in a boundless immensity. The question arises: Why should the Creating Hand solidify a few suns and planets into this single pin-head when there is room for an infinite number of such pin-heads? What *is* our physical universe, a single grain of sand suspended in a shoreless ocean?

If Einstein includes space in his calculations, then one cannot but marvel at the absence of logic in a mind which is otherwise so keenly logical. We are, in that case, required to imagine a universe of space and matter which at certain points comes to an abrupt end. Now, we may fool ourselves into believing that we can imagine such a universe. But that same imagination will insist on peeking beyond the boundaries which it, itself, has imposed upon the universe. It will stubbornly inquire whether or not something exists beyond. The inevitable answer to such inquiries is that space, at the very least, is to be found on the other side of the little fence, or wall, or hedge, which our imagination has built around the universe. Of course, we can continue forever to build that imaginary little fence, or wall, or hedge. But we will also continue forever to find at least more space beyond it. In other words, space is infinite.

Now, I am well aware that this last statement is open to criticism. It may be argued that space is not absolutely infinite; that it lacks the existence of the so-called Stellar universe from being an absolutely infinite existence. Space (in this magazine) does not permit me here to show that the statement in question is ultimately correct, and as I intend, further on, to hint at the unity of our apparently heterogeneous universe, I will satisfy the critical mind at present with the following modified statement: The universe of matter and space is infinite. Or, to put it differently: existence in its totality is infinite.

Here, we have, I believe, a truth which will serve as a foundation, beyond comparison solid, for the little structures of Truth which we every once in a while attempt to build. In the first place, it is a self-evident truth; as self-evident as the truth that the shortest line between two points is the straight line. In the second place, it con-

cerns the nature of the whole of existence instead of that of an infinitesimal part of the whole. And when we have obtained an idea of the nature of the whole, the nature of the part and that of the relationship between the part and the whole cannot forever remain an unfathomable mystery.

We have erred in the past either by assuming that the universe is finite or else by ignoring its infinite nature. The intellect will not reach its ultimate aim of understanding solely through concentrating on the study of the part or of a group of parts. It should be guided by the realization that a whole exists, and that the nature of that whole is naturally and inevitably infinite. Such a realization comes easily to him who is able to live in thought beyond the limits of his immediate surroundings and personal interests, beyond the limits of earth and sun-system in the center of a universal vastness. For it is the endlessness of this vastness which will suggest to him the possibility of ultimately solving some of the deeply hidden mysteries of existence in its depths.

To return to the presence of space, which led us to the conclusion that existence in its totality is infinite: a curious fact concerning it is, that the mind *cannot think it away*. Imagination is capable of scattering the beauties of the flower, of destroying the sun, of obliterating the starry hordes. But the most elastic imagination is absolutely incapable of destroying the existence of boundless space. It takes away that illimitable vastness merely to find that another infinite vastness has taken its place. Space is indestructible.

Now, instead of endeavoring to imagine its destruction, let us try to imagine its creation. The mind meets with the same peculiar difficulty. It is absolutely incapable of conceiving a condition of no-space. Space is uncreated. Apparently then space is eternal in its existence. The self-evidence of this truth rather forces itself upon our minds, provided of course that we think about space at all. And it seems to have forced itself upon the minds of the ancients who took its presence, or rather, omnipresence, for granted. The Babylonians, for instance, caused Apsu and Tiamat to create an army of monster-gods who peopled the "waters of the deep." But they did not so much as hint at the origin of the "deep." The deep was conveniently there. And in Genesis, Jahveh created the heaven and the earth, and darkness was on the face of the deep. Again the deep existed before the dawn of creation.

UMI

We have come face to face with eternity on another occasion, *viz.*, in our laboratory. It was discovered there that matter is indestructible, and that its amount in the universe is for ever the same. Of course, when we accept the story of creation we are forced to conclude that the universe of matter was destined to exist eternally after a certain moment. Prior to that moment it did not exist. This is palpably absurd. It is just as ridiculous to conceive of infinite duration with a beginning as it is to conceive of infinite space with a starting point. That which will exist eternally in the future has existed eternally in the past. It is a strange immortality that needs to be born!

When we state therefore that matter is indestructible, we clothe in different words the sublime fact that the universe of matter in its totality is absolutely eternal. The whole never changes, the parts do. Now, it will be agreed that the existence of two eternities is a logical impossibility. It follows then that the eternity of matter is identical with that discovered in space. A single eternity pervades matter and space alike. And when two seemingly separate existences are both eternal, they are fundamentally one and the same existence. Our apparently heterogeneous universe of matter and space is a unit—a single existence of a uniform fundamental nature.

This last conclusion follows immediately from the fact that something exists which is infinite in its totality. We have seen that the totality of existence is boundless. This being so, there can be no question of several separate and independent existences composing the universe. The single thing which is infinite as a whole is the only thing that can possibly exist. Its presence excludes the possibility of the existence of one or more things beyond the infinity of that presence. Space and the stellar universe are solidly linked by the bond of an ultimately identical fundamental substance.

Pascal's brilliant definition of the universe may therefore be modified as follows: The universe, an infinite—eternal sphere of substance, the center of which is everywhere and the circumference nowhere. I shall not here attempt to discuss the possible nature of this fundamental "substance." It is merely my aim in this article to show that a little thought on space may reveal the fact that enormous possibilities of truth lie concealed in its depths. Nor do such truths differ greatly from the truths concerning existence which we hitherto have accepted on faith by failing to derive them from fact and reality.

Let me suggest that infinity, omnipresence and eternity are to be definitely located in the universe. They *are* existence in its totality. In the light of cosmic philosophy such attributes emerge from the abstract into the real. In the past we have paid too much attention to ourself, to its future existence in a hereafter, and to the Power that determines the nature of that future existence. We possessed a deity principally because we were in need of one. We erected his throne anywhere beyond the supposed limits of this mortal world, and endowed him with the attributes of infinity and eternity. But these attributes were meaningless to us, even as our Supreme Power, though we intuitively realized the necessity of his existence, could be associated with reality only with the utmost difficulty.

All this is due to the fact that evolution had not as yet coaxed man from his shell of self-centeredness. On the whole, man lives in a little universe of his own making; a little universe bounded by the limits of self-interest. What lies beyond these limits, lies also beyond the limits of his intellectual powers. How many of us, to-day, look at the stars without seeing them? How often do we not babble learnedly about the infinite and the eternal without suspecting in the least that we daily face these realities; without suspecting that we live, move and have our being *in* them?

However, evolution constantly urges man to live in an ever-broadening universe. Contact with life wears off some of the hardness of that shell of self-centeredness. Experience, and principally that experience which is the product of struggle, adds little by little to the size of the world in which he lives. History teaches nothing but just that. It records man's struggles with nature, and subsequently with organized society. It records the gradual development of the mind and a corresponding growth of the universe which the mind is contemplating. Less thought of self, and more thought of the balance of existence; less selfishness and more altruism; less ignorance and more understanding—here we have in a nut shell the mysterious aims of human evolution.

That man's mind is broadening by leaps and bounds, and that it is beginning to seek divinity in the infinite reality which constitutes his universal home becomes more and more apparent. Perhaps we should go back as far as Columbus and his discovery of the New World to find the birth of the universal mind. Man became compelled to think in terms of continents instead of in terms of shacks and palaces, in terms of earths instead of in terms of private estates

Steamships, railroads, telegraph and telephone, wireless, flying machine—are they not the creations of a mind that has learned to fly away from home, from ME, and do they not in turn teach the average man to think thoughts that are broad and universal rather than narrow and local?

Copernicus, Galileo, Newton, and their illustrious followers paved the way for the cosmic mind, drew the attention of man to the existence of a stupendously large universe, which he could not very well exclude from his philosophic calculations. The ultimate outcome will be, that divinity will be sought in the one reality beyond which nothing exists, *viz.*, existence in its infinite totality. This statement may at present sound to many ridiculous, if not unreligious. This may be due to the fact that we are not accustomed to live in thought in an infinite universe. Also, we are on the whole rather well satisfied with the deity which the ancients so conveniently conceived for us. But, a little reason will convince us that many objections from a religious point of view might be eliminated. Let us forget, for the moment, the appearance of the universe. Let us forget that such a base, miserable thing as matter is to be found in it. Let us further consider that this infinite is inseparably associated with eternity . . . But, enough! Let the reader think for himself, and decide for himself.

In concluding, it should be remarked that the infinite presents us with a number of truths that are helpful in bridging the wide chasm which seems to separate the part from the whole, the finite from the infinite. Starting from the proposition that existence in its totality is infinite, the following truths suggest themselves:

1. That which is infinite is also eternal.

1*a*. An infinite whole is eternally unchangeable (as a whole).

2. There can exist but one infinite and therefore but one eternity.

3. Given an infinite existence, no other separate and independent existence is possible.

4. An infinite existence is an infinite unit.

5. All individual existence is fundamentally part of the infinite existence.

6. An infinite whole consists of an infinite number of constituent parts.

7. That which is infinite is eternally unchangeable as a whole, but its parts are necessarily subject to constant change.

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